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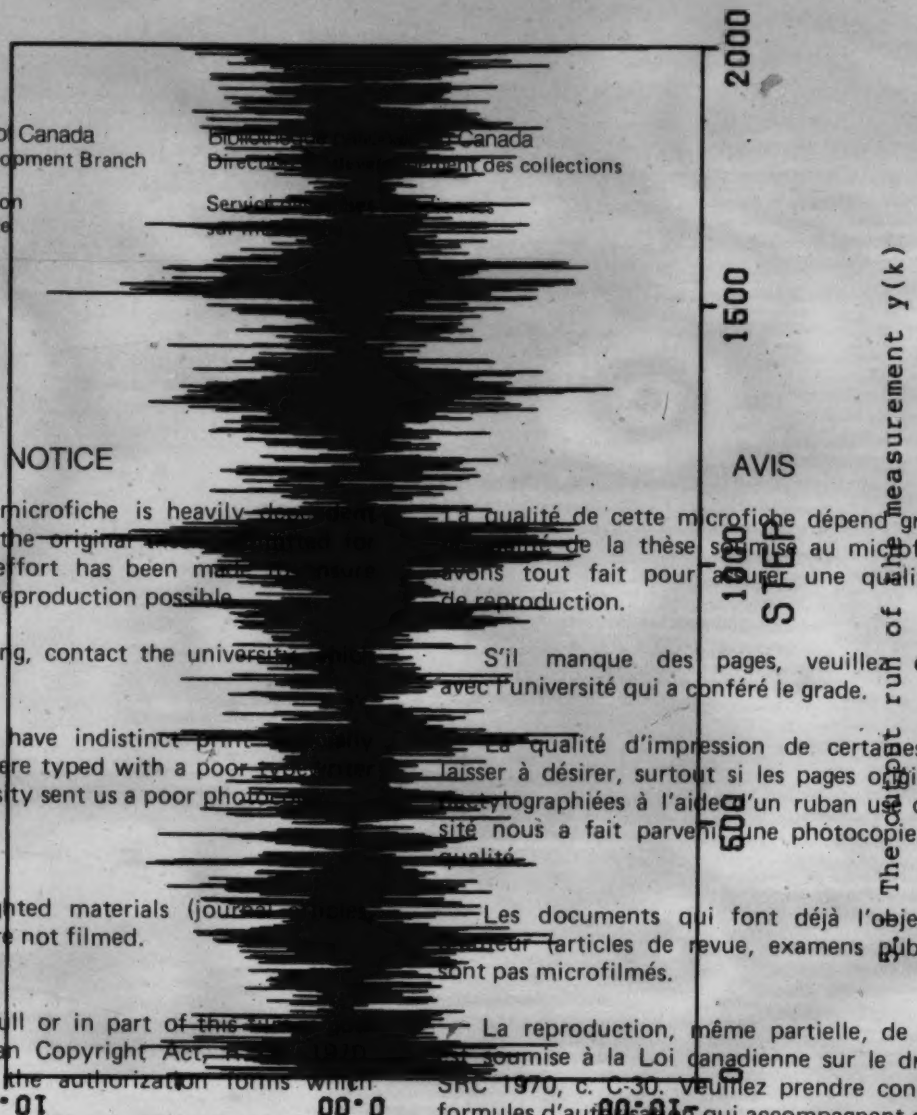

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Full Name of Author — Nom complet de l'auteur

Chang Gun Kim

Date of Birth — Date de naissance

Jan. 2, 1948

Country of Birth — Lieu de naissance

Korea

Permanent Address — Résidence fixe

133, Michener Park, Edmonton, Alberta

Title of Thesis — Titre de la thèse

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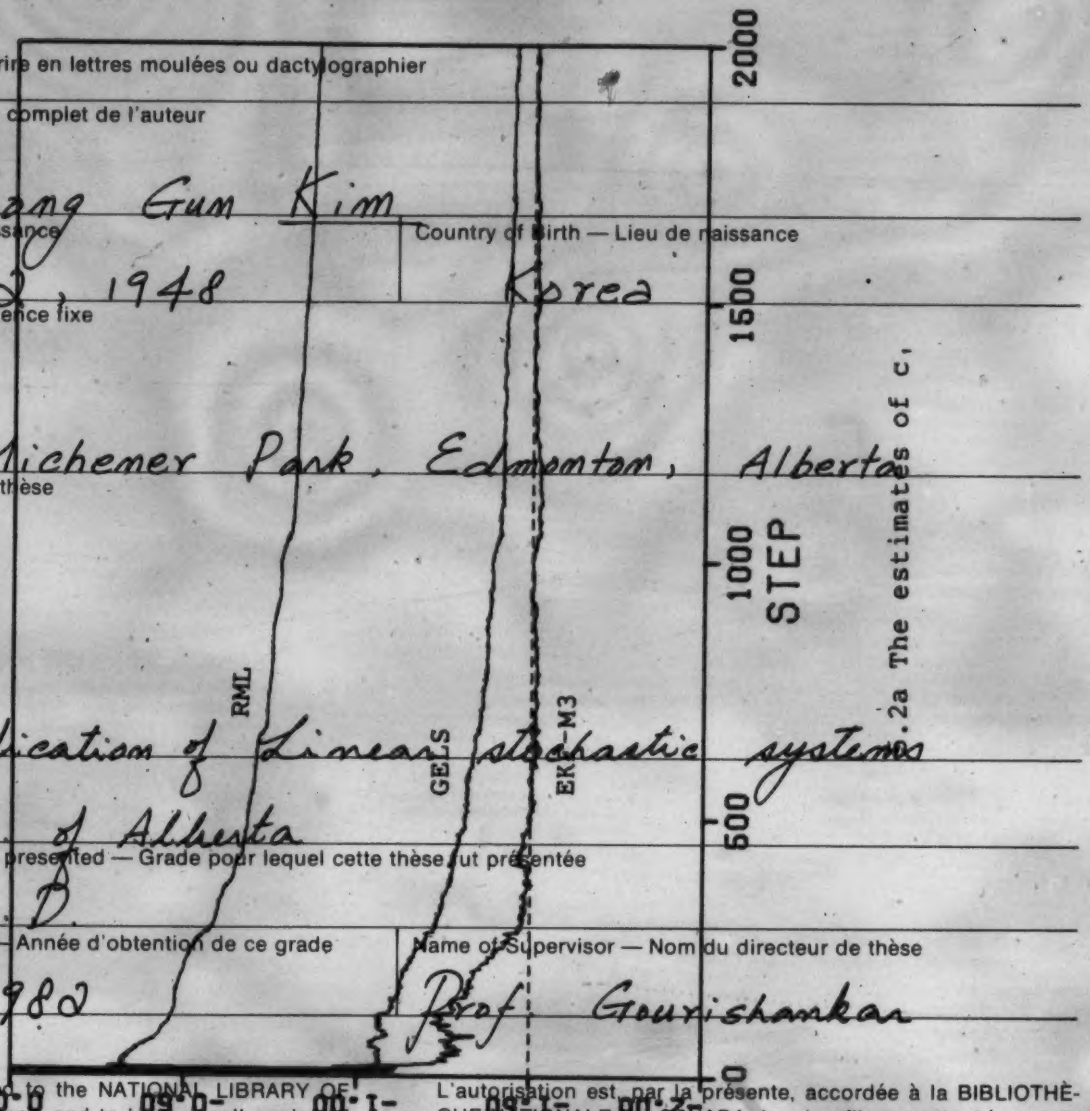
Name of Supervisor — Nom du directeur de thèse

Prof Gourishankar



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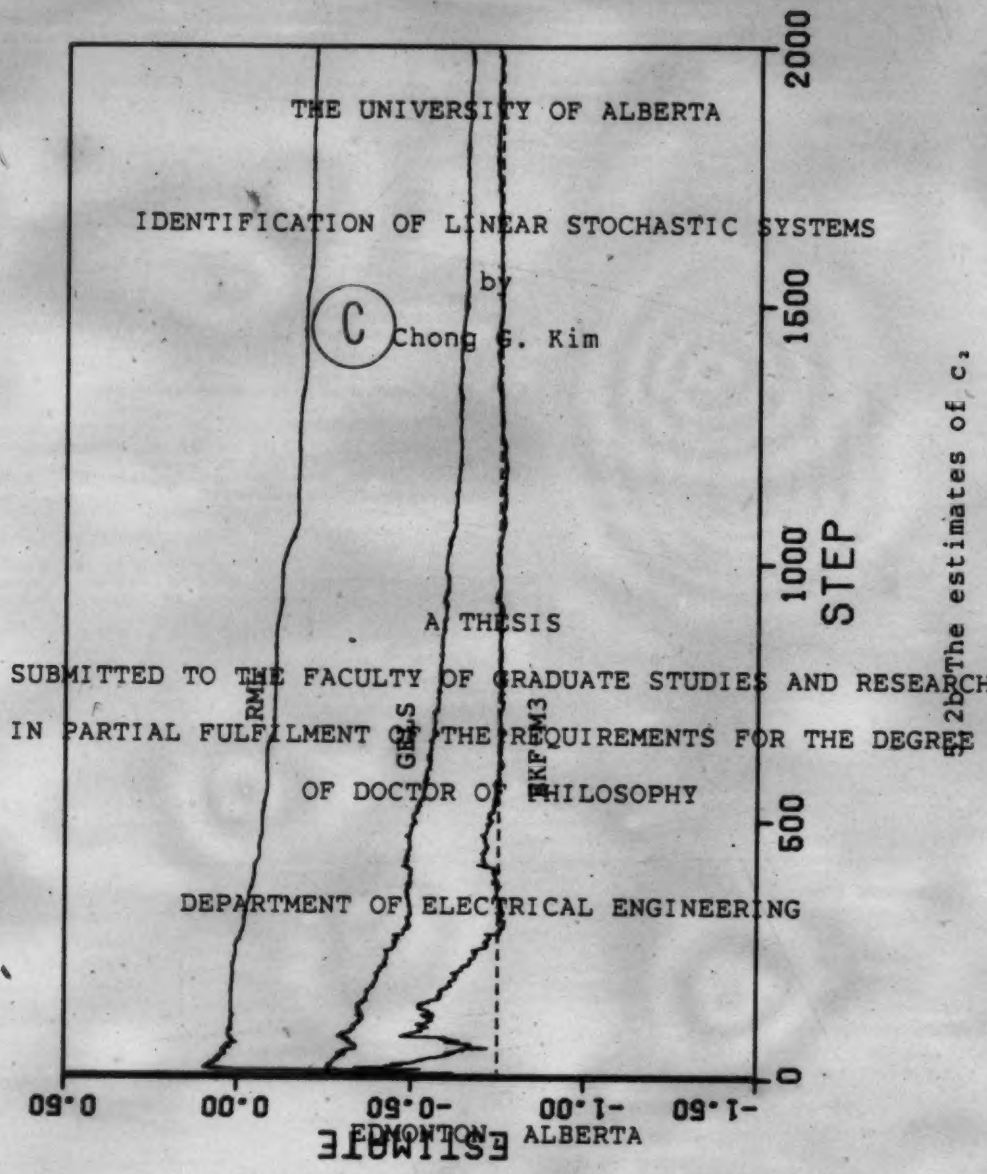

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IDENTIFICATION OF LINEAR STOCHASTIC SYSTEMS

by

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A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH

IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE

OF DOCTOR OF PHILOSOPHY

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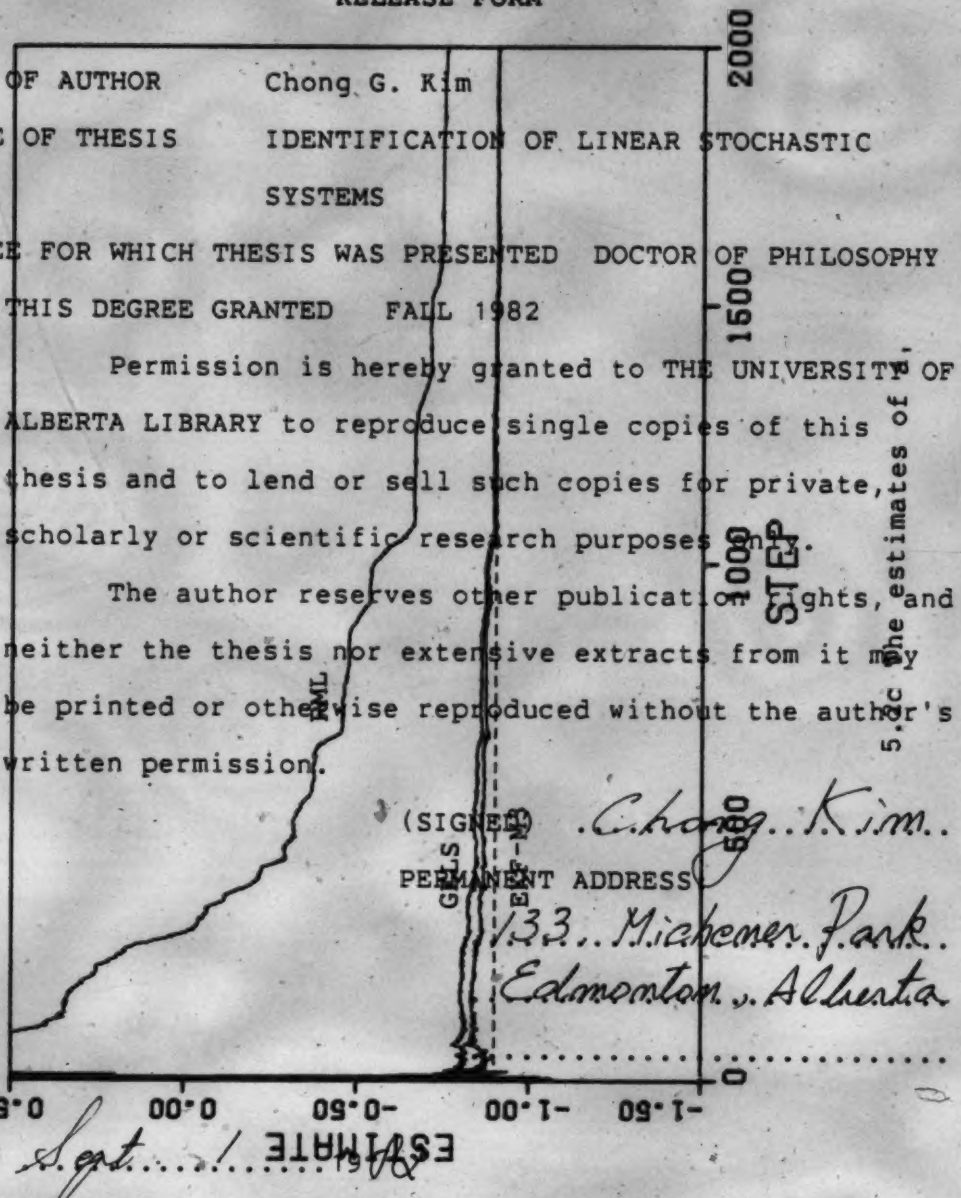

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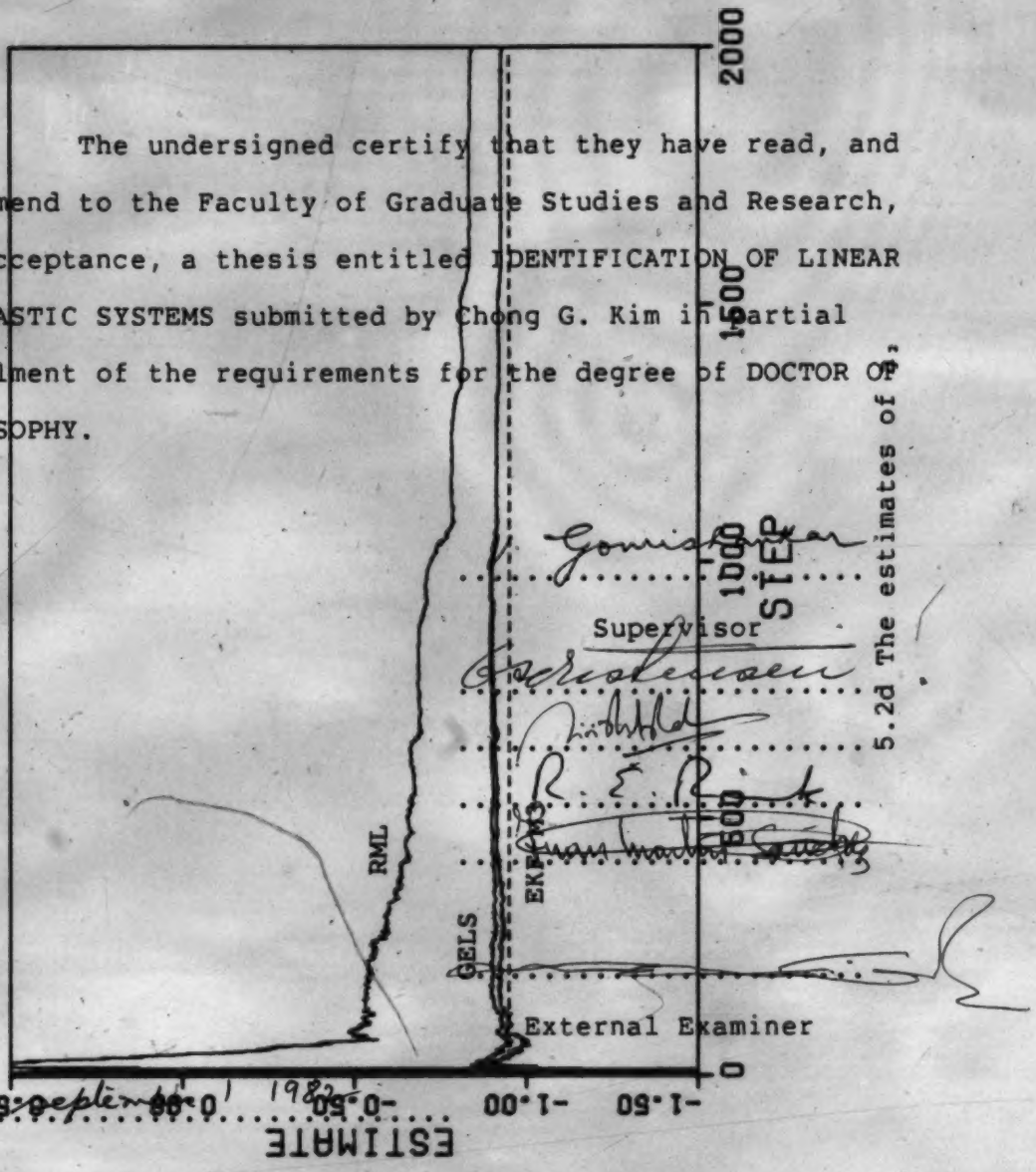


$$x(k+1) = Ax(k) + dv(k)$$

$$y(k) = h'x(k) + v(k)$$

ABSTRACT

where d = [0.570 -0.027 0.306 -0.619]'

In this thesis, a fast algorithm for the parameter thus, c = [0.430 -0.153 0.474 -0.041]'

identification of linear stochastic systems is derived and and v(k) ~ N(0.0, 3.988)

its unbiased convergence property is proved.

Two features of this system are worth noting. This To lay the ground work for the above development, a system is very sensitive to noise (Lee, 1964) and new faster algorithm for linear deterministic system is consequently, it is difficult to identify the system derived by improving the previously published Kudva-Narendra parameters without bias (Saridis, 1974). Also algorithm. Geometric interpretation of the Kudva-Narendra

algorithm helps one to understand how this algorithm works

and consequently to improve the convergence rate thus 2

resulting in a rapidly converging algorithm with reduced is not strictly positive real.

computational requirements.

The same three algorithms used in the first simulation The algorithm for the stochastic system is obtained by study are also used here. For the EKF-M3 and GELS modifying the Extended Kalman Filter algorithm using a algorithms, the following values were assumed as the canonical form of the innovations representation and using à priori informations on the vector parameter c. The same the concepts developed for the deterministic case. This values are also used as the vector f:

development reveals that other existing algorithms which

were developed using different approaches can be shown to be

modifications of the EKF algorithm.

The RML algorithm was also initiated with the same initial

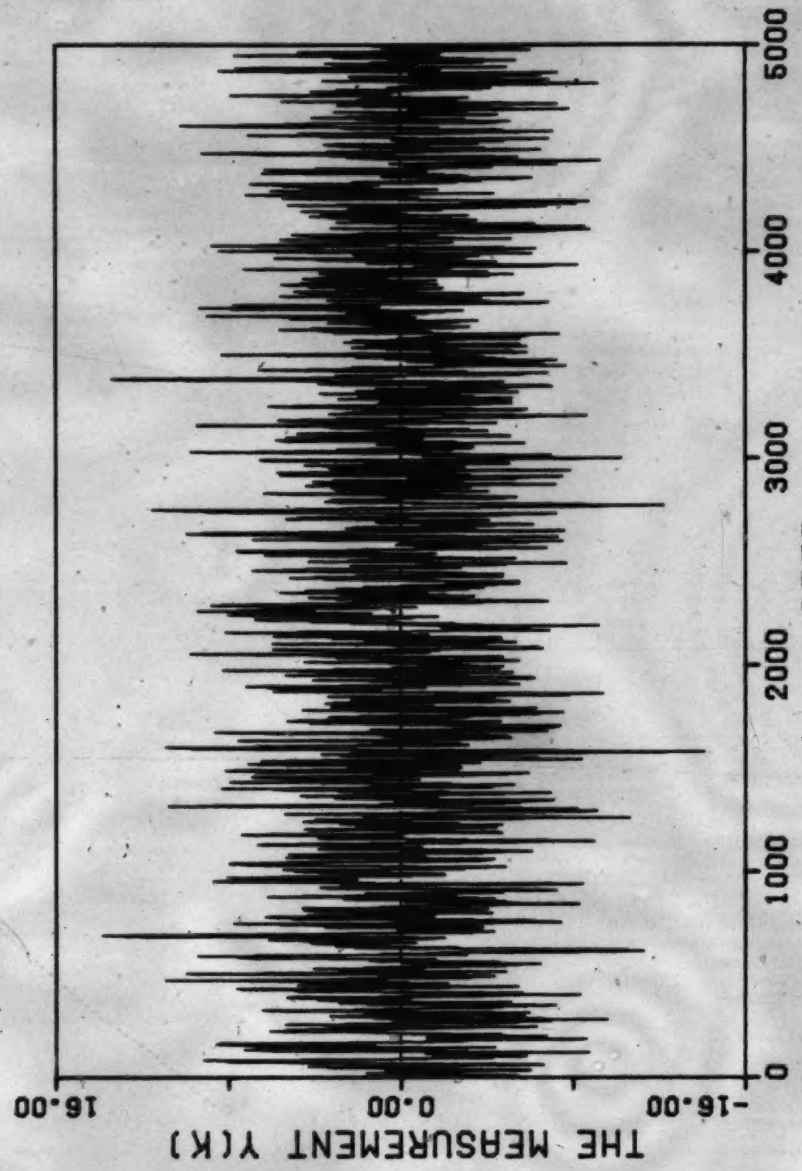
values for fair comparisons. The RML algorithm is compared with

two other existing algorithms by means of simulation previous

studies. These studies show the superiority of the simulation studies are

shown in Figure 5.3 and 5.4. iv

algorithm with respect to convergence rate and computational requirements.



5.3 The output run of the measurement y(k)

ACKNOWLEDGEMENTS

The author acknowledges with a deep gratitude the encouragement and direction from his supervisor Prof. V. G. Gourishankar throughout the research work.

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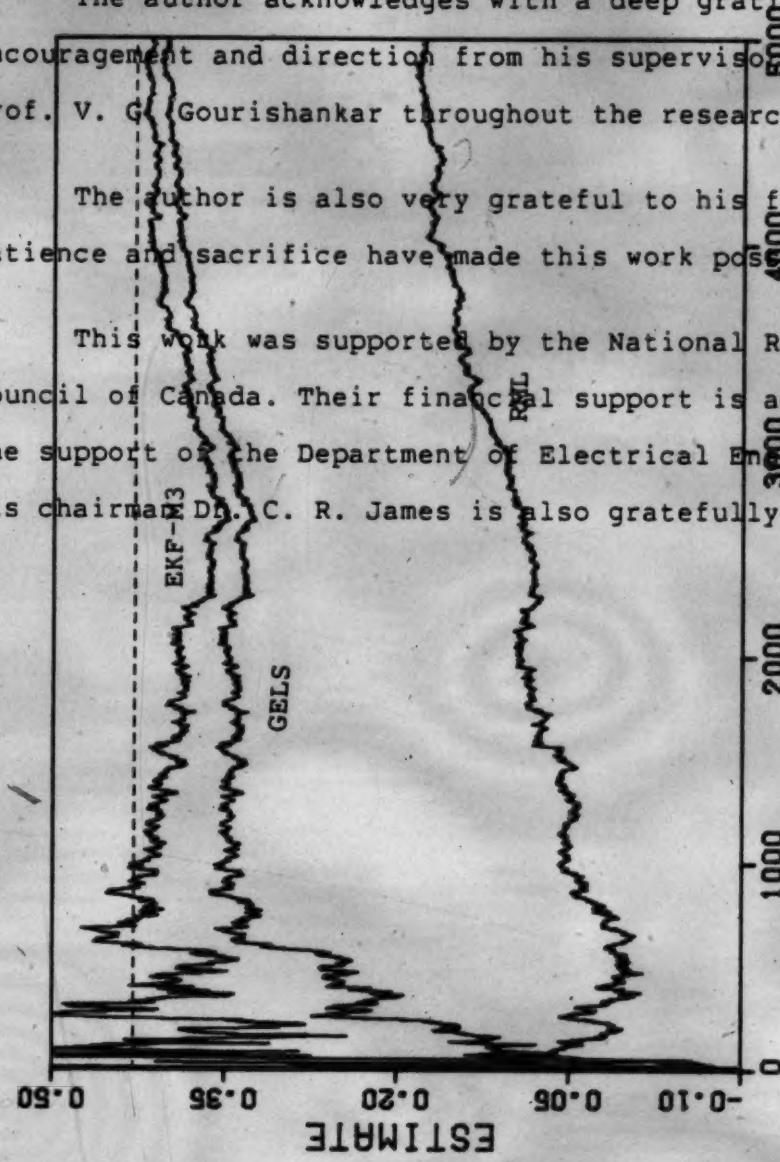

5.4a The estimates of

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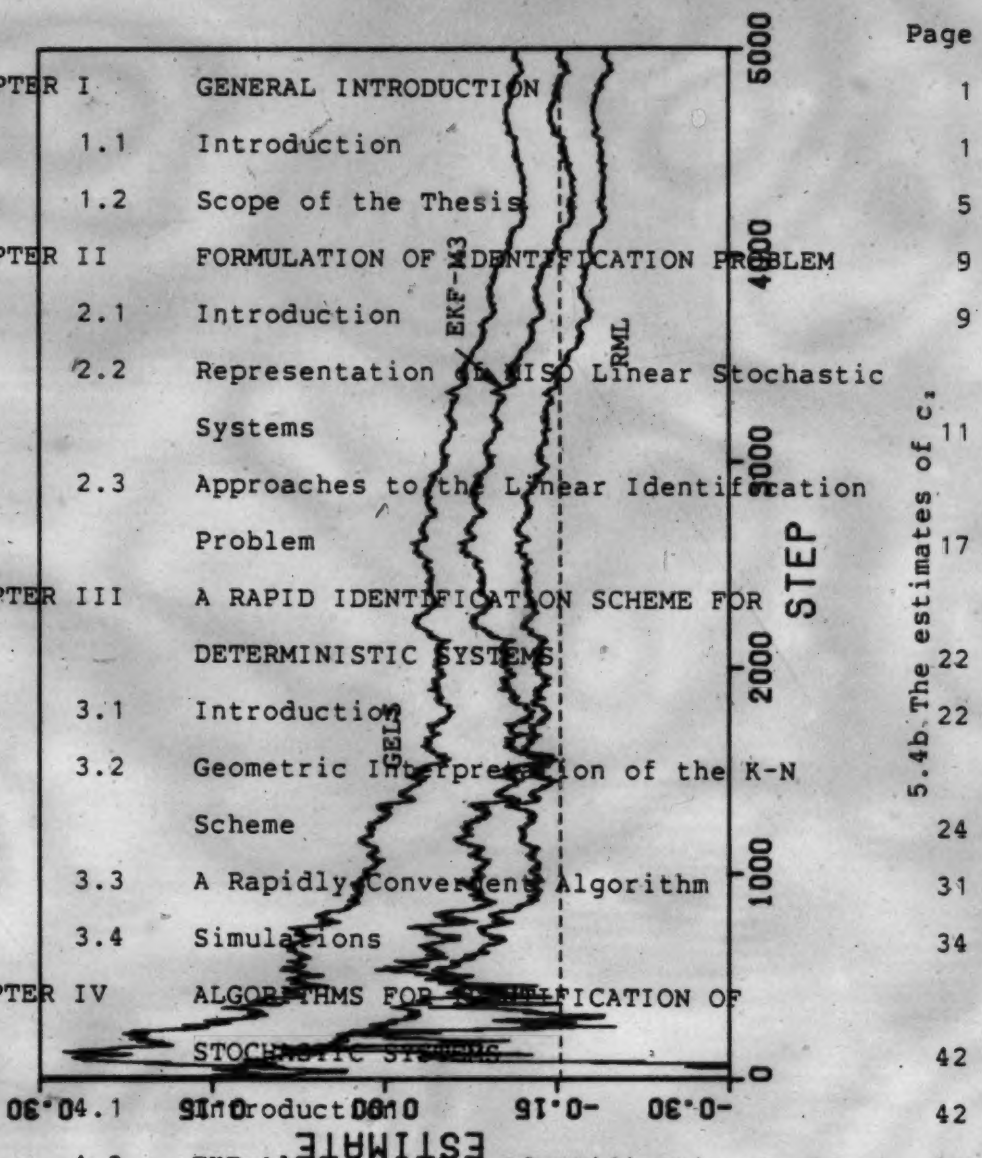

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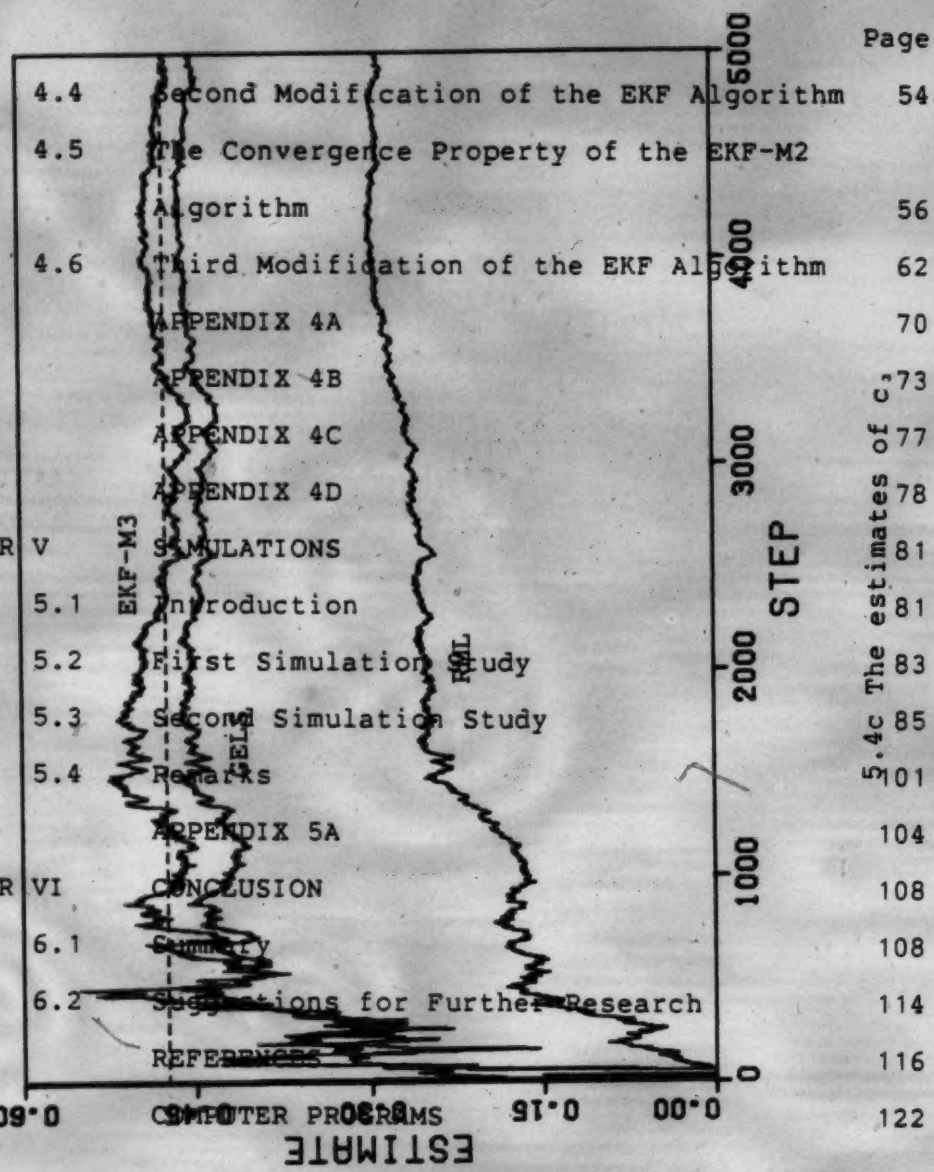

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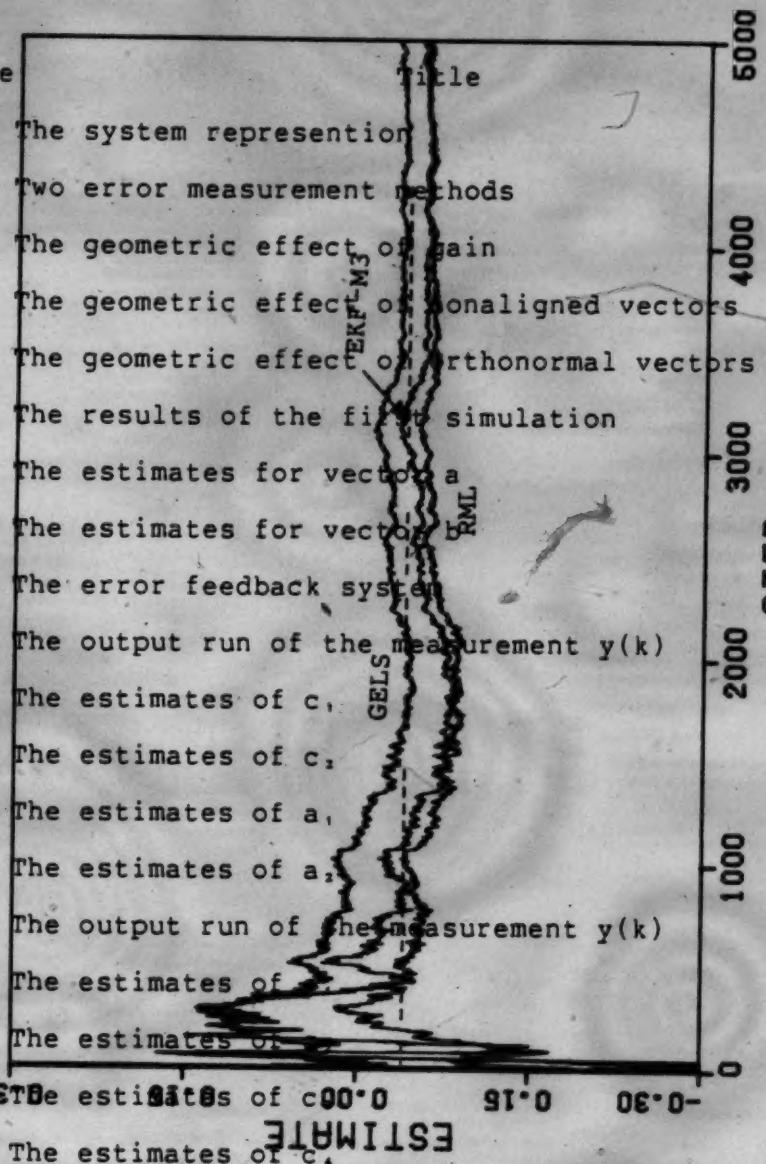

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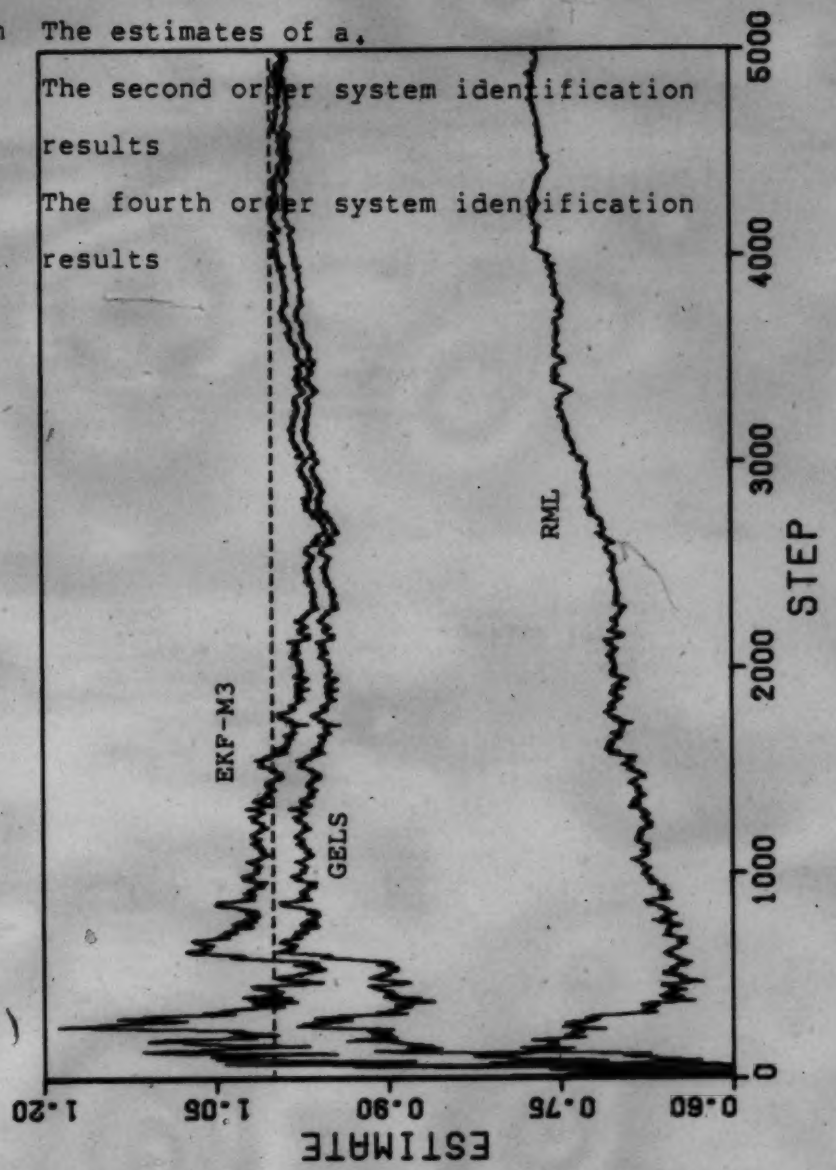


5.4e The estimates of a_1

CHAPTER I

GENERAL INTRODUCTION

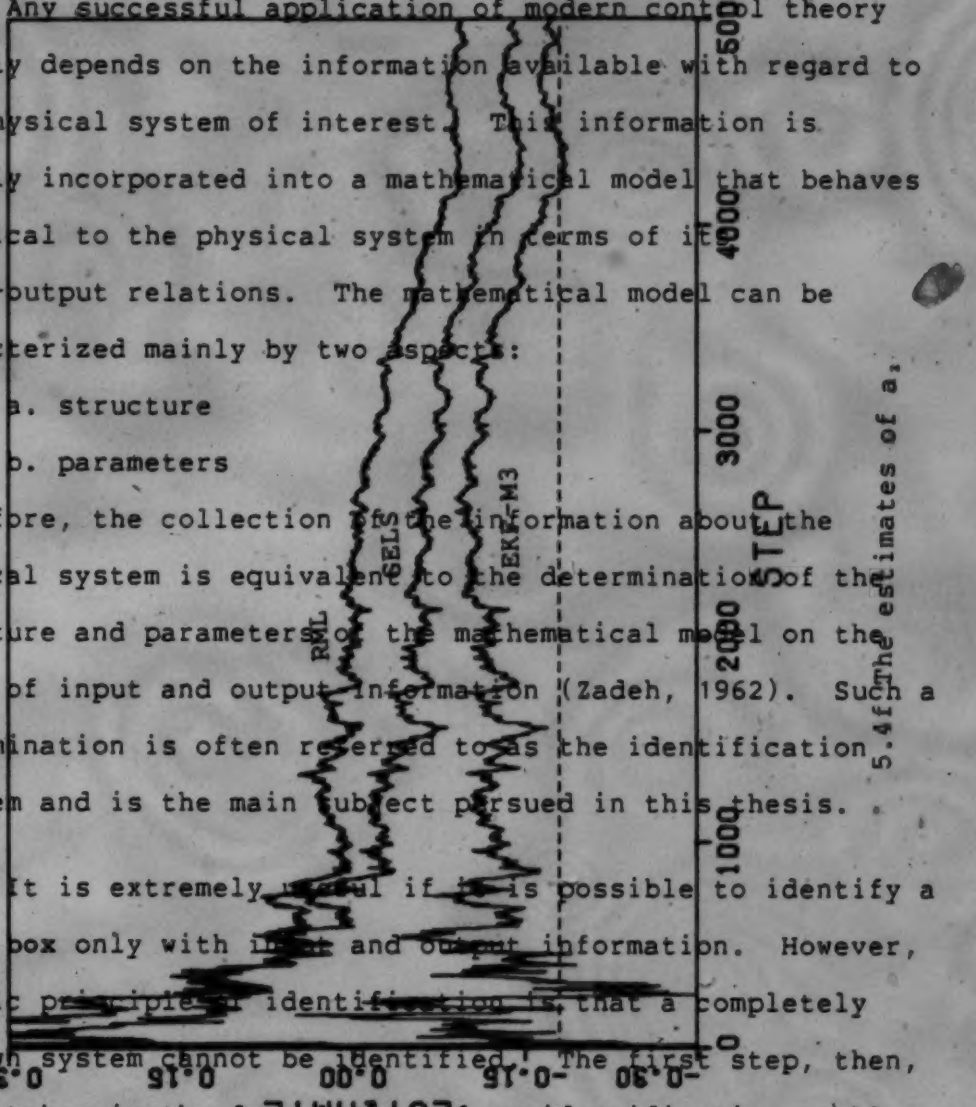
1.1 Introduction

Any successful application of modern control theory largely depends on the information available with regard to the physical system of interest. This information is usually incorporated into a mathematical model that behaves identical to the physical system in terms of its input-output relations. The mathematical model can be characterized mainly by two aspects:

a. structure

b. parameters

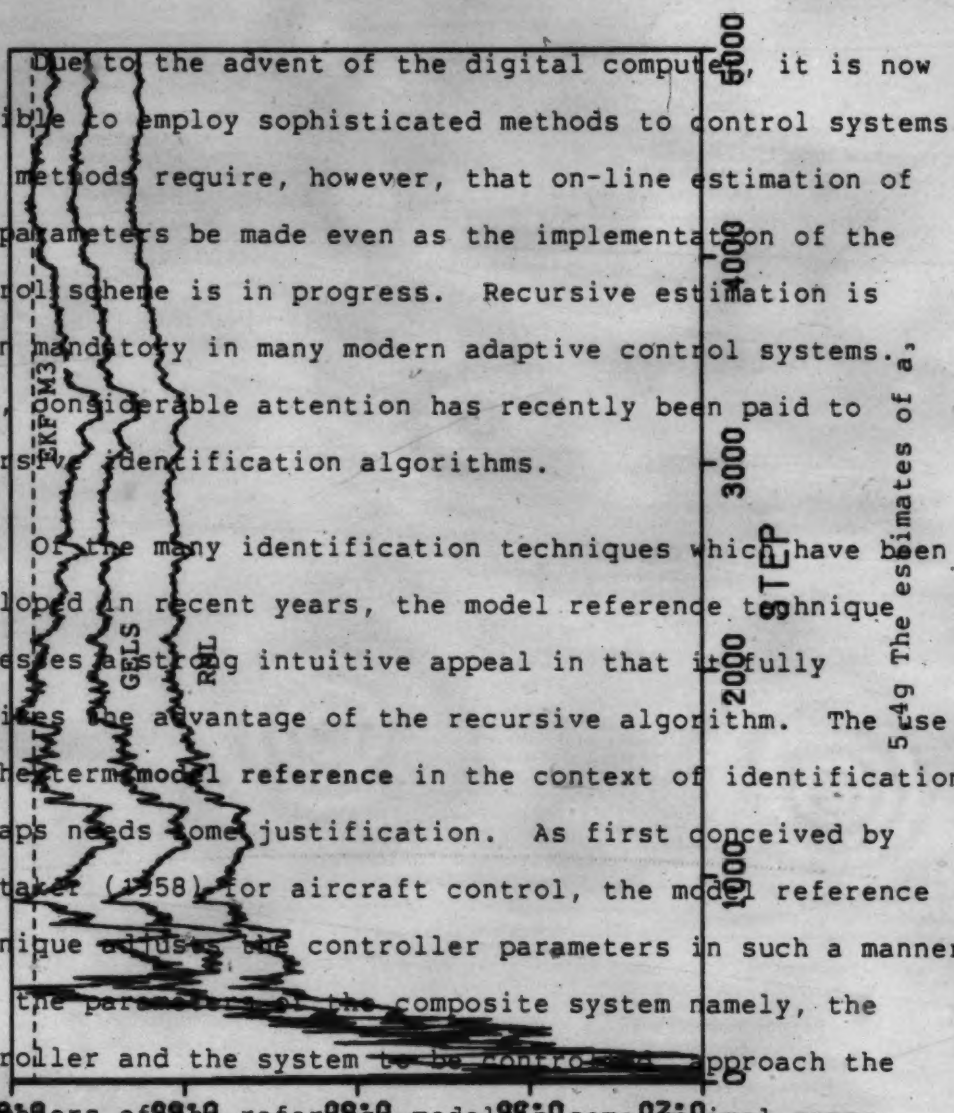
Therefore, the collection of the information about the physical system is equivalent to the determination of the structure and parameters of the mathematical model on the basis of input and output information (Zadeh, 1962). Such a determination is often referred to as the identification problem and is the main subject pursued in this thesis.



It is extremely useful if it is possible to identify a black box only with input and output information. However, a basic principle of identification is that a completely unknown system cannot be identified. The first step, then, to be taken in the formulation of an identification problem is to characterize the system. This characterization, in general, involves the selection of an appropriate structure

for the system. The structure chosen should reflect factors such as, any à priori knowledge of the system and the ultimate purpose of the identification.

Due to the advent of the digital computer, it is now possible to employ sophisticated methods to control systems. Such methods require, however, that on-line estimation of the parameters be made even as the implementation of the control scheme is in progress. Recursive estimation is often mandatory in many modern adaptive control systems. Thus, considerable attention has recently been paid to recursive identification algorithms.

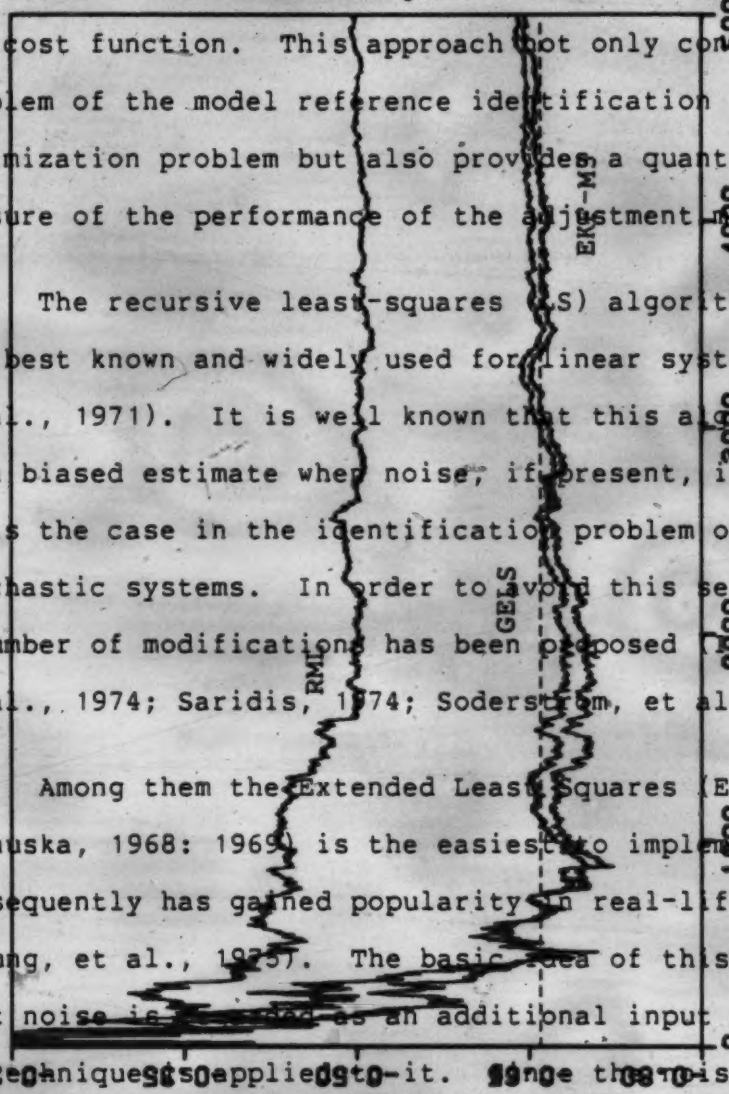


5.4g The estimates of a;

Of the many identification techniques which have been developed in recent years, the model reference technique possesses a strong intuitive appeal in that it fully utilizes the advantage of the recursive algorithm. The use of the term model reference in the context of identification perhaps needs some justification. As first conceived by Whittaker (1958) for aircraft control, the model reference technique adjusts the controller parameters in such a manner that the parameters of the composite system namely, the controller and the system to be controlled approach the parameters of the reference model in some optimal sense. The philosophy of this technique can be used for the identification problem by adjusting the model parameters to follow the parameters of the unknown system, which is looked upon as the reference system. This adjustment is best done

in a recursive fashion. A natural way to come up with a suitable recursive algorithm would be to introduce a cost function in terms of the error between the model and the system, and to devise an adjustment mechanism for minimizing the cost function. This approach not only converts the problem of the model reference identification into an optimization problem but also provides a quantitative measure of the performance of the adjustment mechanism.

The recursive least-squares (LS) algorithm is perhaps the best known and widely used for linear systems (Astrom, et al., 1971). It is well known that this algorithm leads to a biased estimate when noise, if present, is correlated as is the case in the identification problem of linear stochastic systems. In order to avoid this serious drawback a number of modifications has been proposed (Hastermann, et al., 1974; Saridis, 1974; Soderstrom, et al., 1978).



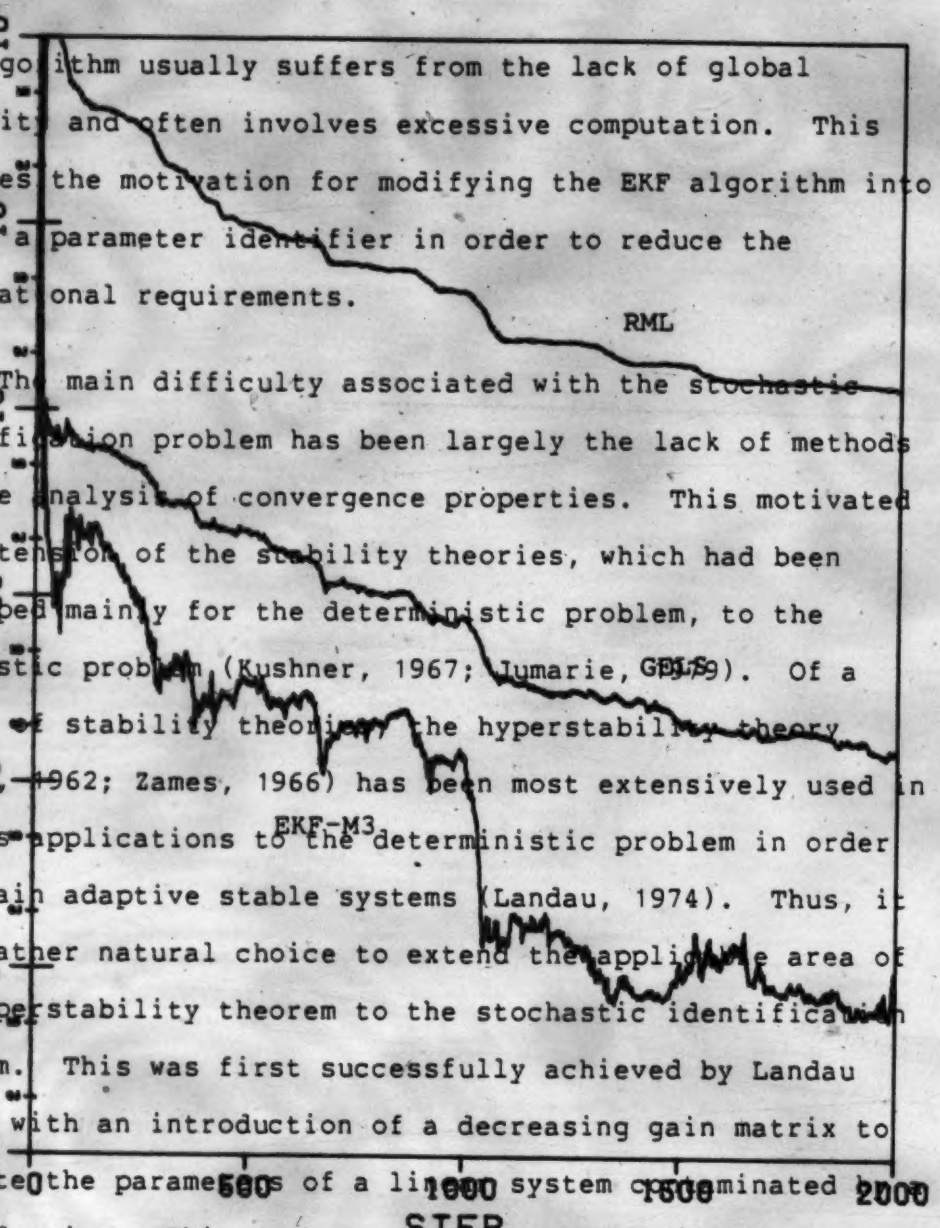
5.4h The estimates of a

Among them the Extended Least Squares (ELS) algorithm (Panuska, 1968; 1969) is the easiest to implement and consequently has gained popularity in real-life applications (Ljung, et al., 1975). The basic idea of this approach is that noise is treated as an additional input signal and the LS technique is applied to it. Since the noise is not available for measurement, it is replaced by previous residuals. Early simulation studies seemed to indicate that this method had good convergence property (Panuska, 1968; Kashyap, 1974). But a study carried out by Ljung, et. al

(5995 Remarks, however, has revealed that, contrary to prior simulat The results, of the ELS compared consistent summarized converge t Since are 5 values 5-6 of the parameters sin The convergence is dependent on the convergence properties of the algorithms. p The required estimation is nonconvergence, which happening sterms by the initial squared errors by providing same weight can be obtained by the introduction of a prefilter involving the unknown parameters. Approximating the

Both studies show that the performance of the EKF-M3 prefilter recursively with the updated estimates results in was vastly superior to those of the other two algorithms for the recursive approximate maximum likelihood (RML) method almost all the the parameters. The performance of the RML (Soderstrom, 1973). This algorithm was developed by an algorithm on the other hand varied considerably depending on approximation of the off-line maximum likelihood (ML) method the parameters. For example, in the second study its (Astrom, et al., 1966) and it possesses a nice local convergence rates for estimates $c_1(k)$ and $c_3(k)$ were convergence property. No global convergence results are, unacceptably slow while those for $c_2(k)$ and $c_4(k)$ were however, available at this time (Panuska, 1980b). surprisingly fast. Though the overall convergence rate of

the RML by the inclusion of the unknown parameters in its s algorithms was shown to be as fast as the system becomes nonlinear s experienced by the more general identification problem. As a result of the system becomes the convergence of this estimation problem considerably as compared to that of ELS Among the numerous approaches for convergence of the local EKF-M3 estimation problem, the Extended Kalman Filter (EKF) both approaches are quite similar as shown in Figure 3.2 with the only 1) gradients the EKF approach reflected terms in the both at each step after the same around the previous estimates and the application of the Kalman filtering theory (Kalman, 1960),



5.5 The second order system identification results

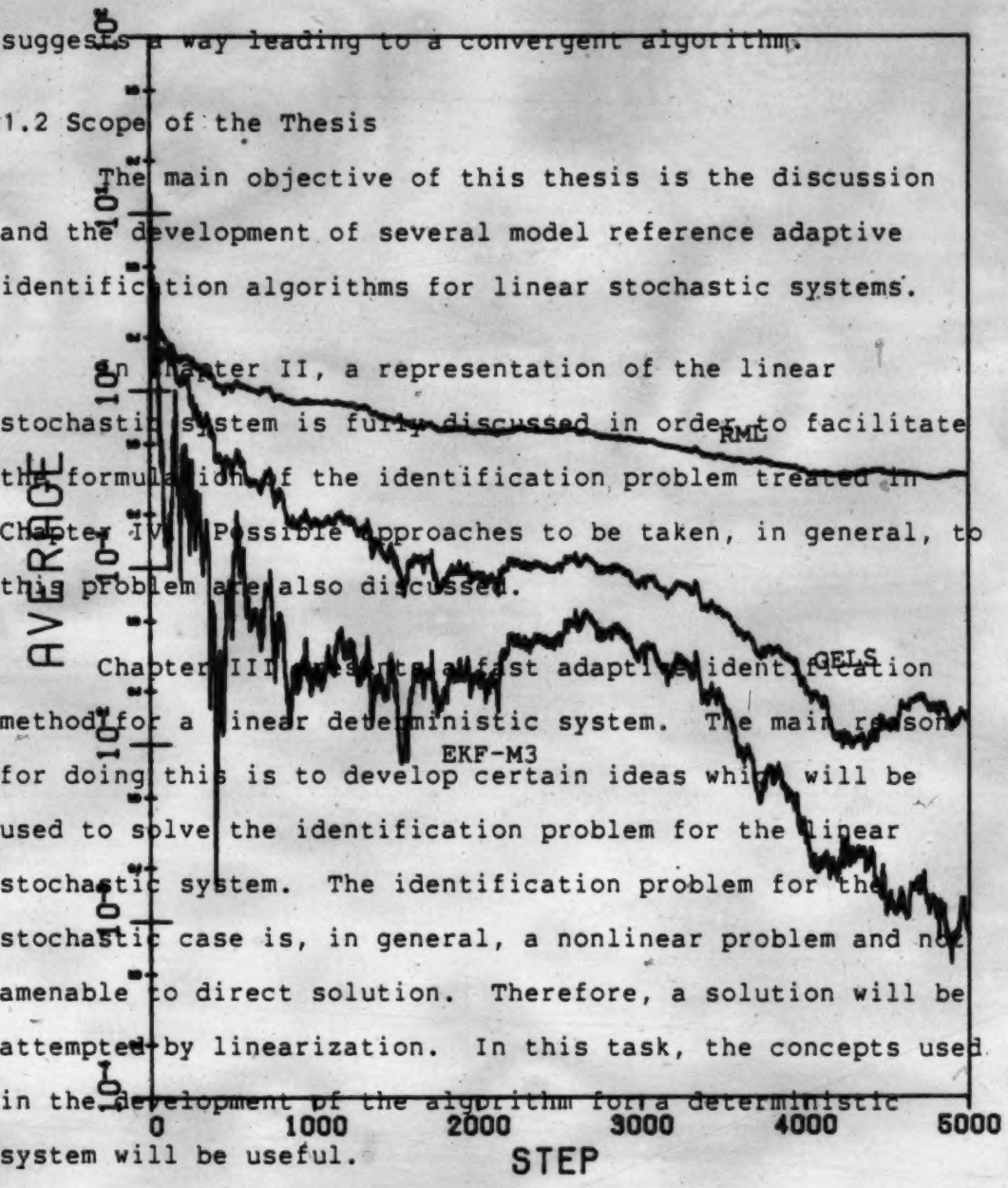
suggests a way leading to a convergent algorithm.

1.2 Scope of the Thesis

The main objective of this thesis is the discussion and the development of several model reference adaptive identification algorithms for linear stochastic systems.

In Chapter II, a representation of the linear stochastic system is fully discussed in order to facilitate the formulation of the identification problem treated in Chapter IV. Possible approaches to be taken, in general, to this problem are also discussed.

Chapter III presents a fast adaptive identification method for a linear deterministic system. The main reason for doing this is to develop certain ideas which will be used to solve the identification problem for the linear stochastic system. The identification problem for the stochastic case is, in general, a nonlinear problem and not amenable to direct solution. Therefore, a solution will be attempted by linearization. In this task, the concepts used in the development of the algorithm for a deterministic system will be useful.



In Section 3.2, a geometric interpretation of the Kudva-Narendra scheme (Kudva, et al., 1974) is given. This approach, which is very crucial to the development of the fast algorithm, provides greater insight into and consequently a clearer understanding of the K-N scheme. In

5.6 The fourth order system identification results

APPENDIX 5A

Derivation of the Generalized Extended Least Squares Algorithm (GELS)

Chapter IV deals with the linear stochastic identification problem. Since this identification problem is inherently a nonlinear estimation problem, approximation techniques are usually called for. Among the many techniques available, the EKF technique is of particular interest because of its simplicity and thus popularity in real applications. Section 4.2 discusses the application of the EKF to the identification problem. In an attempt to reduce the computational requirements which arise in this approach, modifications of the EKF algorithm are made resulting in two simplified algorithms. These are given in Section 4.3 and 4.4. For obvious reasons these are labelled as EKF-M1 and EKF-M2, respectively. Though these two algorithms have similarities to some existing ones to the author's best knowledge, the exact algorithms reported here have not appeared in the literature.

Convergence analysis is probably one of the most important means to assess the usefulness of identification algorithms. However, since EKF-M1 turns out to be similar to the RML algorithm, no detailed discussion of its convergence property is included here. In Section 4.5,

As was pointed out in Section 4.5, it should be noted that the difference between the ELS algorithm and EKF-M2 algorithm is that the ELS algorithm uses the exact $\hat{v}_1(k)$ while the EKF-M2 algorithm uses a posteriori error $y(k) - \hat{y}(k)$. It should also be mentioned that the EKF-M2 algorithm was obtained by generalizing EKF-M1 using a canonical form which included a particular vector as a special case. This generalization will now be used to obtain a Generalized Extended Least Squares algorithm. The GELS algorithm is then of the form

$$z_2(k+1) = F' z_2(k) + h y(k) \tag{A1.1}$$

$$\hat{\theta}_1(k+1) = F' \hat{\theta}_1(k) + h[y(k) - \hat{y}(k)] \tag{A1.2}$$

$$\hat{p}(k+1) = \hat{p}(k) + P_{22}(k)\phi(k) m_3(k)[y(k) - \hat{y}(k)] \tag{A1.3}$$

$$y(k) = \phi(k)' \hat{p}(k) \tag{A1.4}$$

$$\phi(k) = [\hat{\theta}_1(k)', z_2(k)']' \tag{A1.5}$$

$$P_{22}(k+1) = P_{22}(k) - P_{22}(k)\phi(k) m_3(k) \phi(k)' P_{22}(k) \tag{A1.6}$$

$$m_3(k) = [1 + \phi(k)' P_{22}(k) \phi(k)]^{-1} \tag{A1.7}$$

$$P_{22}(0) = P_{22}(0)' > 0$$

Covergence of GELS

The errors $\tilde{p}(k) = p - \hat{p}(k)$ and $\tilde{\theta}_1(k) = \theta_1(k) - \hat{\theta}_1(k)$ satisfy the following error equations:

$$\tilde{\theta}_1(k+1) = F_c'\tilde{\theta}_1(k) - h\gamma(k) \quad \text{(A2.1)}$$

$$\epsilon(k) = \tilde{\theta}_2'\theta_1(k) + \gamma(k) + v(k) \quad \text{(A2.2)}$$

$$\tilde{p}(k+1) = \tilde{p}(k) - P_{22}(k)\phi(k)m_3(k)\epsilon(k) \quad \text{(A2.3)}$$

$$\gamma(k) = \phi(k)'\tilde{p}(k) \quad \text{(A2.4)}$$

$$P_{22}(k+1) = P_{22}(k) - P_{22}(k)\phi(k)m_3(k)\phi(k)'P_{22}(k) \quad \text{(A2.5)}$$

attention is therefore focused on the convergence property of EKF-M2. In Section 4.6, a generalization of EKF-M2 leading to a third algorithm EKF-M3 is discussed. Comparison of the features of EKF-M3 with those of two other existing algorithms, namely, the RML method (Soderstrom, 1973) and the ELS method (Panuska, 1968; 1969).

The usefulness of the EKF-M3 algorithm, the GELS algorithm as well as the RML and ELS algorithms, is evaluated by means of simulation studies. For the purpose of comparison, simulation studies are carried out with the RML and ELS algorithms. These results are presented in Chapter V. Chapter VI provides a summary of the thesis, conclusions and suggestions for further research.

As was mentioned in Section 4.5, the GELS algorithm is shown to converge to the origin without the explicit noise v(k) in the equation (A2.2), one can easily conclude by similarity of the structure of the error equations that the GELS algorithm will yield unbiased estimates of the parameters.

Consequently, the overall unbiased convergence of the GELS algorithm can be established by examining the stability of the error feedback system without the noise term v(k) in equation (A2.2). This stability problem can be stated as follows:

Lemma: Given the following time-varying system:

$$\tilde{\theta}_1(k+1) = F_c'\tilde{\theta}_1(k) - h\gamma(k) \quad \text{(A3.1)}$$

$$e(k) = \theta_2'\tilde{\theta}_1(k) + \gamma(k) \quad \text{(A3.2)}$$

$$\tilde{p}(k+1) = \tilde{p}(k) - P_{22}(k)\phi(k)m_3(k)e(k) \quad \text{(A3.3)}$$

$$\gamma(k) = \phi(k)'\tilde{p}(k) \quad \text{(A3.4)}$$

$$P_{22}(k+1) = P_{22}(k) - P_{22}(k)\phi(k)m(k)\phi(k)'P_{22}(k) \qquad \text{(A3.5)}$$

$$P_{22}(0) = P_{22}(0)' > 0$$

$\tilde{\theta}_1(k)$ and $\tilde{p}(k)$ converge to zero as $k \to \infty$

if $\phi(k)' P_{22}(k)\phi(k) \leq \delta$ for all k where δ is a positive scalar and

$$H(q^{-1}) = 1/2 - \theta_2'(qI - F_c')^{-1}h$$

is strictly positive real.

Proof:

Consider the following quadratic function as a candidate for the Lyapunov function:

$$V(k) = V_1(k) + V_2(k) \qquad \text{(A4)}$$

$$V_1(k) = \tilde{\theta}_1(k)'\Sigma\tilde{\theta}_1(k) \qquad \text{(A5)}$$

$$V_2(k) = \tilde{p}(k)' P_{22}(k)^{-1}\tilde{p}(k) \qquad \text{(A6)}$$

It follows from equations (A3.1-2):

$$\begin{aligned} \nabla V_1(k) &= V_1(k+1) - V_1(k) \\ &= \tilde{\theta}_1(k)'[F_c\Sigma F_c' - \Sigma]\tilde{\theta}_1(k) - 2\tilde{\theta}_1(k)'F_c\Sigma h\gamma(k) \\ &\quad + h'\Sigma h\gamma(k)^2 \end{aligned} \qquad \text{(A7)}$$

$$\begin{aligned} \nabla V_2(k) &= V_2(k+1) - V_2(k) \\ &= \gamma(k)^2 - 2\gamma(k)\varepsilon(k) + \phi(k)' P_{22}(k)\phi(k)m(k)\varepsilon(k)^2 \end{aligned} \qquad \text{(A8)}$$

In equation (A8), the following matrix inversion lemma has been used:

FORMULATION OF IDENTIFICATION PROBLEM

2.1 Introduction

An identification problem is characterized by three quantities (Zadeh, 1962): a class of model, a class of input signals and a criterion. The selection of the class of models generally leads to the determination of the structure that mathematically represents the system. Since input signals have a profound effect on the performance of the identification scheme some limitations are usually imposed on input signals. It has been shown (Aoki, et al., 1970; Yuan, et al., 1977), that the **persistent excitation** condition of input signals is sufficient in many cases to obtain consistent estimates or improve the performance of the identification scheme. However, the selection of input signals itself is the subject of an interesting but separate area of research, hence we will not pursue this matter any further.

The structure of models describing the system under consideration wields considerable influence on the complexity of identification schemes in various ways such as computational requirements, consistency and uniqueness of estimates, etc. Since different identification methods are necessary for different representations there are very few general rules available by which the optimality of the model

structure chosen can be determined. A convenient (A9)

characterization of the model for the recursive model
Since $H(q^{-1}) = 1/2 - \theta_2'(qI - F_c')^{-1}h$ is s.p.r., there exist
reference approach to the identification problem can be made
a real vector ξ, a real scalar τ and a real positive
through the parametric representation since this is
definite matrix Σ such that (Hitz, et al., 1969)
relatively easy to implement on computers.

$$F_c\Sigma F_c' - \Sigma = -\xi\xi' \tag{A10}$$

In the parametric representation of linear systems,

$$F_c\Sigma h = -\theta_2 + \xi\tau \tag{A11}$$

the choice of model structure by which the system is

$$h'\Sigma h = 1 - \tau^2 \tag{A12}$$

represented is influenced by the number of unknown
parameters to be identified and the applicability of the
With these relationships, equation (A7) becomes
real-time optimization techniques which are readily

$$\nabla V_1(k) = -[\theta_1(k)'\xi + \gamma(k)\tau]^2 + 2\gamma(k)e_1(k) - \gamma(k)^2 \tag{A13}$$

available now. The number of parameters to be identified is
thus,
directly related to the efficiency of the identification
algorithm as well as the uniqueness of the solution because
there exists a limit to the number of the parameters which can be
uniquely determined by the input-output relations. Since
Since $\phi(k)$ is bound and $P_{22}(k)$ is nonincreasing, there
the identification problem is often formulated as an
exists a positive scalar δ such that
optimization problem by introducing a cost function, the
applicability of the available optimization techniques (A15)
should be taken into account in the choice of a model
in which case $\nabla V(k) \leq 0$,
structure.
and the feedback system (A3) is stable.

Consequently, the GELS algorithm is assured to produce
In this chapter we restrict our discussion to linear
unbiased estimates of the parameters. The system
multiinput single output (MISO) stochastic systems for the
for the
sake of simplicity. Since a linear multiinput multioutput
convergence of GELS compared to SKF will be
(MIMO) system can be regarded as a group of MISO systems
this additional restriction (A5)
each of which has an input-output pair associated with it,
the extension to MIMO systems can be readily carried out.

CHAPTER VI

CONCLUSIONS

6.1 Summary

This thesis is entirely devoted to the development of various recursive identification methods for deterministic as well as stochastic discrete linear time-invariant systems. Though at the beginning of each chapter, there was a brief discussion on the specific subject which was dealt in that chapter, it is the purpose of this section to summarize the salient features of each chapter and the results obtained in this thesis.

In Chapter II, a canonical form of the innovations representation for the multiinput singleoutput system is introduced. The preference for using the innovations representation over other representations, in particular for the identification problem is largely related to the uniqueness of parameter estimates for a given measurement. This uniqueness, which is of great importance for the identification problem, originates from the fact that the innovations representation is the sole minimum phase system for a given rational spectral density (Anderson, et al., 1979). This, in turn, implies that for a given measurement $y(k)$, a stationary process, there exists only one minimum phase system whose output could be regarded as the measurement and this minimum phase system is identical to

2.2 Representation of MISO Linear Stochastic Systems

Kalman (1963) has shown that only the completely controllable and completely observable part of a system can be characterized from input-output relations. Thus, the system that is of interest in this thesis is one which is completely controllable and completely observable. The phase variable representation (Chen, 1970) of such a system is:

$$x(k+1) = Ax(k) + bu(k) + \omega(k) \tag{2.1a}$$

$$y(k) = h'x(k) + \nu(k) \tag{2.1b}$$

where $x(k)$ is an $n\times 1$ state vector,

$u(k)$ is an input variable,

$\omega(k)$ is an $n\times 1$ system noise,

$y(k)$ is an output variable,

$\nu(k)$ is a measurement noise,

$$A = \left[\begin{array}{c|c} a & \begin{matrix} I \\ \hline 0 \end{matrix} \end{array}\right]$$

$h = [1\ 0\ \cdots\ 0]'$,

$a = [a_1\ a_2\ \cdots\ a_n]'$,

$b = [b_1\ b_2\ \cdots\ b_n]'$,

I is the unity matrix of appropriate dimension,

and $'$ denotes transpose of a matrix.

Here $\omega(k)$ and $\nu(k)$ are assumed to be $N(0,Q)$ and $N(0,r)$, respectively. By means of Kalman filtering theory (Kalman,

the innovations representation. Thus, by adoption of the innovations representation, a risk of ambiguities of identified parameters could be almost eliminated. Furthermore, the structure of the canonical form introduced is suited especially to the least squares method for the identification problem. This form is also flexible enough to facilitate the development of new identification algorithms.

Chapter III presents a fast identification algorithm for the deterministic linear system. This was done by geometrically interpreting the Kudva-Narendra (K-N) identification algorithm and improving the convergence speed by the introduction of orthonormal vectors. The special advantage of this scheme is that the overall computational requirements are considerably reduced in comparison to the K-N algorithm. While the underlying idea of this development is simple and straightforward, the geometric interpretation makes it possible to visualize how the K-N algorithm works and consequently to improve the convergence rate. Furthermore, this approach provided a basis for the direction to be taken for the development of identification algorithms when noise is present, i.e. the stochastic system identification which is the subject of Chapter IV.

A stumbling block in the use of the standard least squares method for the stochastic system identification problem is, in general, the inclination of the least squares

1960) the number of the unknown parameters which characterize noise can be reduced by an innovations representation (Kailath, 1968; Geesey, et al., 1969; Astrom, 1970) to a form which requires a total of only 3n+1 unknown parameters to completely characterize the n-th order system to be identified against [2n+1]+[n(n+1)/2] unknown parameters in equation (2.1), namely

$$x(k+1) = Ax(k) + bu(k) + dv(k) \tag{2.2a}$$

$$y(k) = h'x(k) + v(k) \tag{2.2b}$$

where $v(k)$ is $N(0,w)$,

$$w = h'ph + r$$

p is the steady-state covariance matrix of the state error of the Kalman filter,

and $d = [d_1 \cdots d_n]$ is the steady state Kalman filter gain.

Since the system matrix A in equation (2.2a) contains the unknown vector a, it is useful to replace the partially known matrix A with F, a known matrix but preserving the same structure of the matrix A for the identification problem.

Equation (2.2) can be thus rewritten as:

$$x(k+1) = Fx(k) + (a-f)x_1(k) + bu(k) + dv(k) \tag{2.3a}$$

$$y(k) = h'x(k) + v(k) \tag{2.2b}$$

method to bias the estimates (Astrom, et al., 1971) if noise is correlated, as is generally the case. Several techniques have been proposed for the recursive identification problem to get around this drawback. The basic philosophy behind the proposed methods in common is to decompose the correlated noise into a sequence of weighted white noise or innovations, which is often called as Wold decomposition and then to use the standard recursive least squares method to identify the augumented parameters which includes the parameters for weighing the innovations. One of the obvious problems arising in this approach is that the innovations are not amenable to measurement. Consequently, different methods have been suggested to estimate the innovations.

Since the identification problem for the linear stochastic system can be considered as a nonlinear estimation problem, the Extended Kalman Filter algorithm has been quite often used as a starting step for developing new algorithms. In chapter IV, this approach is once again taken to derive several different identification algorithms in order mainly to alleviate the unnecessary computational burden. The modifications are made possible because of the following observations:

1) the EKF algorithm for the identification problem is assured of its unbiased convergence in the neighborhood of the true values of the parameters.

2) the use of the innovations representation combined with

The above observation justifies setting some elements of the approximate covariance matrix of the EKF algorithm to be zero, thus leading to simpler algorithms in term of computational requirement.

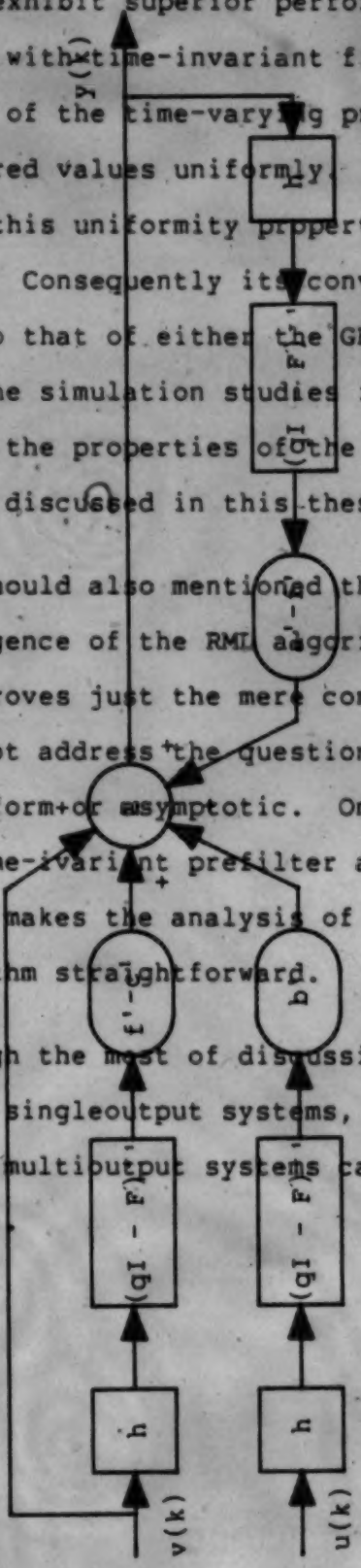
The process of simplification has also revealed that some existing algorithms which were developed by other methods are merely modifications of the EKF algorithm. Thus, this helps to understand better the behaviour of these algorithms. This understanding together with an effective use of the representation form has long been an issue in the adaptive observer problem for deterministic systems (Astrom, et al., 1971; Kudva, et al., 1974; Carroll, 1976; Kreisselmeier, 1977). The apparent advantage of this representation is that the a priori knowledge of the system, the linearity in parameters (Astrom, et al., 1971) so that the usual least-squares method can be used for identification problems (Kudva, et al., 1974).

The differences among the algorithms discussed in this thesis, namely, RML, GELS and EKF-M3 are worth noting. While in the RML and GELS algorithms, à priori errors are used to replace the true innovations, à postériori errors are used for the EKF-M3 algorithm. It is this à postériori error that is thought to be responsible for faster convergence of the EKF-M3 algorithm to the true values compared to the RML and GELS algorithms. Secondly, the RML algorithm requires the knowledge of the time varying model an important requirement to the which the GELS and EKF-M3 is the choice of an appropriate model order. It is known that the parameter estimation algorithms with large varying

prefilter exhibit superior performance compared to algorithms with time-invariant filters, provided the parameters of the time-varying prefilter approach to the final desired values uniformly. In the case of the RML algorithm this uniformity property is not present in the prefilter. Consequently its convergence property is inferior to that of either the GELS or EKF-M3 algorithm as shown by the simulation studies in this thesis. Table 6-1 summarizes the properties of the various recursive algorithms discussed in this thesis.

It should also mentioned that the analysis of Ljung on the convergence of the RML algorithm (Ljung, 1979) though elegant, proves just the mere convergence of the algorithm and does not address the question whether the convergence is either uniform or asymptotic. One of the advantages of using a time-invariant prefilter as in the EKF-M3 algorithm, is that it makes the analysis of the convergence property of the algorithm straightforward.

Though the most of discussions have been for multiinput singleoutput systems, the extension for multiinput multioutput systems can be easily made (Anderson, 1974; 1977).



2.1 The system representation

when the order of models does not match with that of the system to be identified. There are several methods available for choosing an appropriate model order (Anderson, 1962; van den Boom, et al., 1974; Unbehauen, et al., 1974; Soderstrom, 1977; Young, et al., 1980). Since this subject also stands alone as an interesting research project we leave it here without further discussion.

Since the omission of the control input u(k) does not affect the characteristics of equations (2.5) except to decrease the number of unknown parameters, we shall drop the control input u(k) in the sequel for the sake of simplicity, giving rise to a problem of time-series identification.

The formation of a state vector $\theta(k)$ by including all the unmeasurables in equations (2.5) itself to linear equation in the state space as:

$$\theta(k+1) = \Phi\theta(k) + \eta v(k) \tag{2.6a}$$

$$y(k) = [0\ 0\ H_2(k)']\theta(k) + v(k) \tag{2.6b}$$

where

METHOD	ADVANTAGE	DISADVANTAGE	CORRECTION
ELS (Panuska)	Simple	Possible biased convergence Slow	Use of à priori information Use of prefilter
GELS	Global convergence with proper f vector	Time-invariant prefilter Use of à priori error Use of à posteriori error	
EKF-M2	Simple	Possible biased convergence Slow	Introduction of prefilters Use of à priori error
EKF-M3	Global convergence with proper f vector	Time-invariant prefilter Use of à posteriori error	
RML (Soderstrom)	Local convergence	No global convergence Possible biased convergence Slow	Introduction of projectors Use of à priori error
RML with Projector	Local convergence	Large amount of computation Slow	Time-varying prefilter + projector Use of à priori error

Table 6.1 Comparisons of recursive algorithms

$\theta(k) = [\theta_1(k)', \theta_2(k), \theta_3(k)']$

$\theta_1(k) = z_1(k)$

$\theta_2(k) = -c$

$\theta_3(k) = f$

6.2 Suggestions for Further Research

Fast but bias-free identification algorithms for the linear system using a systematic approach have been developed. But the ever-increasing use of on-line identification methods for systems in the various fields substantially expands the range of application of the identification techniques to systems in which more stringent conditions are often encountered; for example it is often necessary to identify a system with very limited à priori knowledge of the system. Thus there is a need to improve the existing algorithms so as to handle this situation satisfactorily.

The employment of a time-varying prefilter instead of a time-invariant one used for the algorithm developed in this thesis seems extremely promising as an immediate improvement, thus eliminating the requirement of à priori information to guarantee unbiased convergence. One very important problem requiring an immediate attention is the unbiased convergence property of the modified form of the algorithm. This problem also demands a comprehensive study of a more general form of representation for the stochastic system to accomodate the time-varing prefilter made.

Recursive identification problems could also be treated as a model tracking problem. It then becomes natural to introduce the model reference adaptive (MRA) method (Landau, 1974) as a design tool. In this approach

$$\Phi = \begin{bmatrix} F' & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \qquad \Gamma = \begin{bmatrix} h \\ 0 \\ 0 \end{bmatrix}$$

$$S = \begin{bmatrix} 0 & I & 0 \\ I & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

and 0's are understood as vectors or matrices of appropriate dimension.

2.3 Approaches to the Linear Identification Problem

Most of the approaches to the recursive identification problem of linear stochastic systems have been developed from statistical considerations and many of them are based on the modification of the LS method. Among them the best known are the RML method (Soderstrom, 1973), the recursive instrumental variable (IV) method (Young, 1974), the recursive prediction error (PE) method (Moore, et al., 1979: 1980) and the EKF algorithm (Kopp, et al., 1963; Cox, 1964). The main shortcoming of these methods, though some progress has been recently made (Ljung, 1977b), is the difficulty encountered when attempts to analyze the convergence properties of the corresponding algorithms are made.

Another approach which has a great potential is to treat the linear stochastic system as an unknown infinite impulse response (IIR) filter and use the standard least

squares method. This approach will involve the development of an efficient recursive least squares method for an infinite dimensional system and a fast algorithm to transform a polynomial of ever-inceasing order into a rational polynomial with a fixed order.

While this thesis has been devoted to the theoretical development of efficient recursive identification algorithms for stochastic systems, it is appropriate to mention that these algorithms have a wide application in the area of process control (Isermann, ed; 1980), communications (Friedlander, 1982) and processing of seismic data (Lion, 1966). The equation error method† has been widely used for the problem of adaptive observers and schemes which obtained from geophysical exploration to mention a few. work satisfactorily in real applications have been reported (Luder, et al., 1974). The popularity of this method for a the particular problem is largely due to the ease with which the gradient of the quadratic cost function of the errors can be obtained and the simplicity of designing an recursive algorithm to decrease the cost function asymptotically. The results of this method, however, are known to be biased if correlated noise is present. For this reason, the applicability of this method is somewhat limited for stochastic problems. The output error method, on the other hand, has an appealing potential in that it has been shown

Two configurations related to the MRA technique could be laid out depending on the error measurement methods (Robinson, 1957; Mendel 1977; Mahalanabis, et al., 1981).

More extensive computer simulations also needed for a more complete evaluation of the algorithms treated in this thesis. A comprehensive assessment of the performances characteristics including sensitivity to errors in estimation system order and the effect of unusually large impulse noise etc. which are of extreme importance in real applications should be made by means of simulating studies before implementation.

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†The equation error method is often referred as the series-parallel approach and the output error method the parallel approach. These terms apparently have their origin in the configurations used for error measurement (Martin-Sanchez, 1976).

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to yield an unbiased estimate even in the presence of

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correlated noise (Landau, 1976, 1978). Research has since concentrated on the possible extension of this method to the problem of the identification (Dugard, et al., 1980) as well as to adaptive control of stochastic systems. However, for stochastic systems it turns out that the conditions imposed for the overall stability are not always possible to be satisfied because of the involvement of the parameters that are to be identified (Ljung, 1977a, Egardt, 1980, Johnson, 1980).

Fig. 2.2 shows the block diagrams for two error measurement methods

where $A(q^{-1}) = 1 + a_1 q^{-1} + \ldots + a_n q^{-n}$ (2.7a)

$$B(q^{-1}) = 1 + b_1 q^{-1} + \ldots + b_n q^{-n} \quad (2.7b)$$

The inclusion of the unknown parameters to the augmented state vector transforms a parameter identification problem into a nonlinear estimation problem. Since the optimal solution to nonlinear filtering problems is not, in general, realizable in the finite dimensional systems (Bucy, et al., 1971), nonlinear filters are, in most cases, obtained by approximate design methods, while the ERF offers the best simplicity in implementation, thus, being most widely used among the numerous methods, the application of this approach often unnecessarily increases the computational requirements if only the parameters need to be known. It would appear to be worth while to attempt a

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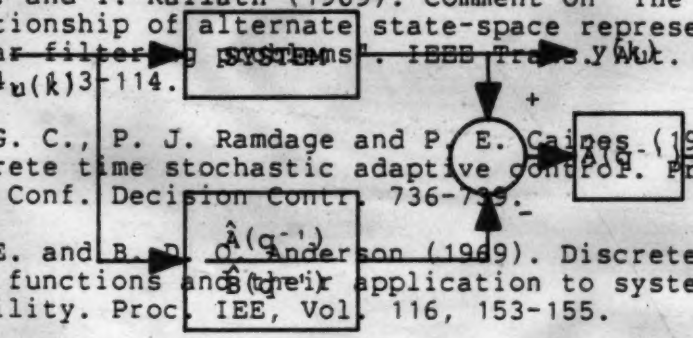

a) The equation error measurement

b) The output error measurement

2.2 Two error measurement methods

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Thus, there is need for increasing the speed of convergence.

This can be achieved by modifying the algorithm (3.1) to

$$\hat{x}(k+1) = C\hat{x}(k) + [\hat{A}(k+1) - C]x(k) + \hat{B}(k+1)u(k) \qquad (3.2)$$

provides a greater insight into the convergence property of the algorithm and consequently a clearer understanding of the scheme. Though the underlying concept is rather simple, this becomes very crucial in the development of an algorithm with rapid convergence property which is important in parameter estimation. This approach also provides a general idea of how the gain of the improved algorithm can be modified to suppress noise. This, in turn, has an enormous implication in the development of an identification algorithm for linear systems developed in Chapter IV.

3.2 Generalized Interpretation of the K-N Scheme

Young (1974) approached the problem of identifying the unknown but constant nxn and nxr, (n≥r) matrices A and B of a time-invariant discrete system given by

$$x(k+1) = Ax(k) + Bu(k) \qquad (3.1)$$

and observed that the following adaptive algorithm

$$\hat{x}(k+1) = C\hat{x}(k) + [\hat{A}(k+1) - C]x(k) + \hat{B}(k+1)u(k) \qquad (3.2)$$

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$$[\hat{A}(k+1),\hat{B}(k+1)] = [\hat{A}(k),\hat{B}(k)] - \frac{\alpha\Sigma[e(k) - Ce(k-1)]\phi(k-1)'}{\lambda\phi(k-1)'\phi(k-1)} \quad (3.3)$$

where C is an nxn stable matrix, i.e. all its eigenvalues lie inside the unit circle,

$\phi(k) = [x(k),u(k)]$,

λ is the largest eigenvalue of Σ

and $e(k) = \hat{x}(k) - x(k)$.

Kudva and Narendra have shown that estimate [A(k),B(k)] will converge asymptotically to [A,B] provided that

i) the gain matrix Σ is chosen to be symmetric and positive definite,

ii) $0 < \alpha < 2$,

iii) the system (3.1) is completely controllable

and iv) the input sequence u(k) is general enough (Kudva, et al., 1974).

It follows from equations (3.1-3) that

$$e(k+1) = Ce(k) + \tilde{P}(k+1)\phi(k) \quad (3.4)$$

$$\text{and} \quad \tilde{P}(k+1) = \tilde{P}(k) - \frac{\alpha\Sigma\tilde{P}(k)\phi(k-1)\phi(k-1)'}{\lambda\phi(k-1)'\phi(k-1)} \quad (3.5)$$

```
      SUBROUTINE GELS(YU,FV,P,ITER,IYU,NORD,NORDD,GAIN,ICH)
C
C     THESE ARE THE GENERALIZED EXTENDED LEAST SQUARES AND
C     EKF-M3 IDENTIFICATION ALGORITHMS FOR THE SYSTEM
C     WITHOUT CONTROL INPUT, i.e. TIME-SERIES, OR WITH
C     CONTROL INPUT.
C     THE CONTROL FLAG 'ICH' SWITCHES BETWEEN GELS AND
C     EKF-M3.
C     THE RESULTS WILL BE STORED IN A TEMPORARY FILE "-GELS"
C     IF GELS IS RUN OR  "-EKF-M3" IF EKF-M3 IS RUN.
C
C     YU:   OUTPUT OF THE SYSTEM WHOSE PAPRAMETERS ARE TO BE
C           IDENTIFIED AND THEN THE CONTROL INPUTS ARE
C           IMMEDIATELY FOLLOWED IF THEY EXIST.
C     FV:   SYSTEM VECTOR OF THE SYSTEM MATRIX F OF THE
C           IDENTIFIER
C
C     P:    THIS VECTOR CONTAINS THE INITIAL ESTIMATES FOR
C           c, a and b VECTORS
C     ITER: THE NUMBER OF ITERARTION DESIRED.
C     IYU:  THE NUMBER OF ENTRIES IN "YU" IF CONTROL INPUTS
C           EXISTS THEN IYU=ITER*2, OTHERWISE IYU=ITER
C     NORD: ORDER OF THE SYSTEM
C     NORDD: 2*NORD IF NO INPUTS
C            3*NORD IF INPUTS EXIST
C     GAIN: SCALAR VALUE FOR THE INITIAL GAIN MATRIX P22
C           P22(0) WILL BE GAIN*IDENTITY MATRIX
C     ICH:  SELECT GELS      IF ICH=0
C                 EKF-M3     IF ICH=1
C
C
      LOGICAL*1 DATA1(8),DATA2(8)
      REAL K(15),X(15),A(15,3),FV(NORD),D(15,3),XI(3),
     $     PRMT(15),YU(IYU),EPSI(3),WKA(3,4),ISIG(3,3),
     $     AMAX(15),AMIN(15),P(NORDD),EPSI0(3)
C
      DATA DATA1/'-','G','E','L','S',3*' '/
      DATA DATA2/'-','E','K','F','-','M','3',' '/
C
C
C     ATTACH THE PROPER TEMPORARY FILE FOR THE OUTPUTS
C
      IF(ICH .EQ. 0)CALL FTNCMD('ASSIGN 3=?;',0,DATA1)
      IF(ICH .EQ. 1)CALL FTNCMD('ASSIGN 3=?;',0,DATA2)
C
C     DETERMINE IF THERE ARE INPUTS TO THE SYSTEM
```

where $\tilde{P}(k) = P(k) - P$,

$\hat{P}(k) = [\hat{A}(k), \hat{B}(k)]$

and $P = [A, B]$

Since the matrix C does not appear in equation (3.5), the convergence of this equation which is the main concern here, does not depend on whether C = 0 or ≠ 0. In simulation studies, C = 0 is the most convenient choice.

In proving the stability of equation (3.5), for the sake of convenience, we institute the following change of variables:

$$F = \begin{vmatrix} FV \\ \hline I \\ 0 \end{vmatrix}^T$$

$$\psi(k) = \phi(k-1)/||\phi(k-1)|| \qquad (3.6)$$

where $||.||$ denotes the Euclidean norm.

Equation (3.5) reduces to:

$$\tilde{P}(k+1) = \tilde{P}(k) - \alpha P(k)\psi(k)\psi(k)^T \qquad (3.7)$$

Since P is a positive definite matrix, there exists a nonsingular matrix T such that

$$T\Sigma T^T = I \qquad (3.8)$$

where Λ is a diagonal matrix whose diagonal elements λ_i lie in the range $0 < \lambda < 1$

```
C
      NDATA=NORDD/NORD
C     Premultiplying equation (3.7) by T and using equation
C
C (3.8), SET SIG(0)=GAIN*IDENTITY MATRIX. THIS IS EQUIVALENT TO
C     SETTING P22(0)=GAIN*IDENTITY MATRIX. THUS, THE INVERSE
C     OF SIG(0), ISIG(0)=IDENTITY MATRIX/GAIN
C
      DO 1 I=1,NDATA
      DO 2 J=1,NDATA
      ISIG(I,J)=0.0
      IF (I .EQ. J) ISIG(I,J)=1.0/GAIN
    2 CONTINUE
    1 CONTINUE
C
C     SET A(0) AND D(0) EQUAL TO ZERO
C
      DO 5 I=1,NORDD
      DO 6 J=1,NDATA
      A(I,J)=0.0
    6 D(I,J)=0.0
    5 CONTINUE
C
C     SET K(0) AND X(0) EQUAL TO ZERO
C
      DO 7 I=1,NORDD
      K(I)=0.0
    7 X(I)=0.0
C
C     SET INITIAL ESTIMATES P,  P=[FV-C,A-FV,B]
C
      DO 8 I=1,NORD
      P(I)=FV(I)-P(I)
    8 P(I+NORD)=P(I+NORD)-FV(I)
C
C     SET INITIAL VALUE OF AMAX AND AMIN EQUAL TO ZERO
C
      DO 9 I=1,NORDD
      AMAX(I)=-100.0
    9 AMIN(I)=100.0
C
C
C
C     If we choose a = 1 for convenience, λ_i must be 1 and
C
C Λ = I.  Furthermore, if we choose T = I for convenience
C
      DO 1000 KKK=1,ITER
      YH=0.0
      L=1
      DO 10 I=1,NDATA
      DO 11 J=1,NORD
      YH=YH+P(L)*X((J-1)*NDATA+I)
   11 L=L+1
   10 CONTINUE
```

$$\tilde{P}(k+1) = \tilde{P}(k) - a\Lambda\tilde{P}(k)\psi(k)\psi(k)' \tag{3.9}$$

where
$$P(k) = T\tilde{P}(k) \tag{3.10}$$

We can partition equation (3.10) row wise as below:

$$\tilde{\rho}_i(k+1) = \tilde{\rho}_i(k) - a\lambda_i\psi(k)\psi(k)'\tilde{\rho}_i(k), \quad i\epsilon[1,n] \tag{3.11}$$

where $\tilde{\rho}_i(k)$ is the i-th row vector of the nx(n+r) matrix $\tilde{P}(k)$, λ_i is the i-th diagonal element of Λ

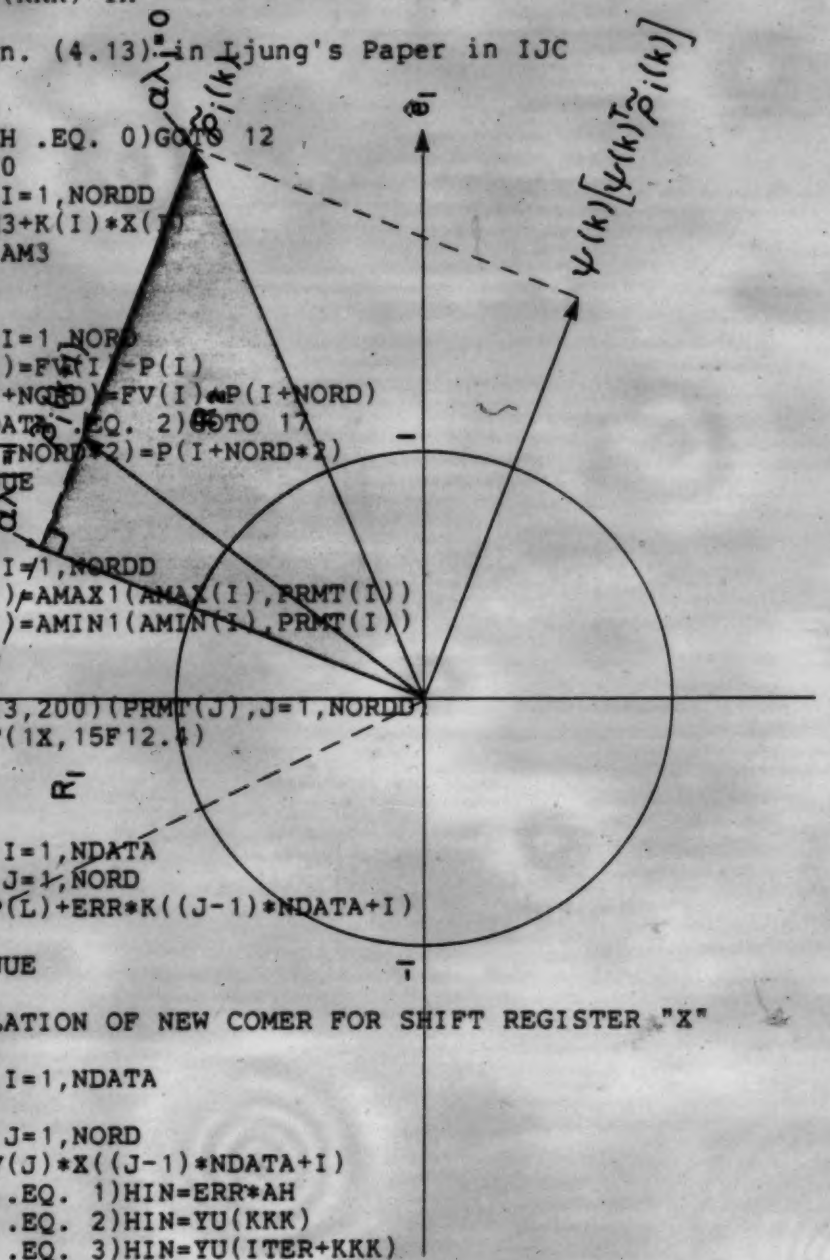
and
$$0 < a\lambda_i < 2.$$

Since
$$\tilde{\rho}_i(k)'\psi(k) = ||\tilde{\rho}_i(k)||\cos\theta_i, \tag{3.12}$$

the choice $a\lambda_i = 1$ makes $||\tilde{\rho}_i(k+1)||$ minimal for a given $\psi(k)$ where θ_i is the angle between $\tilde{\rho}_i(k)$ and $\psi(k)$.

then
$$a\Sigma\lambda_i = 1.$$

```
C
C
      ERR=YU(KKK)-YH
C
C     SEE Eqn. (4.13) in Ljung's Paper in IJC
C
      AH=1.0
      IF (ICH .EQ. 0)GOTO 12
      AM3=0.0
      DO 15 I=1,NORDD
   15 AM3=AM3+K(I)*X(I)
      AH=AH-AM3
C
C
   12 DO 17 I=1,NORD
      PRMT(I)=FV(I)-P(I)
      PRMT(I+NORDD)=FV(I)-P(I+NORD)
      IF (NDATA .EQ. 2)GOTO 17
      PRMT(I+NORD*2)=P(I+NORD*2)
   17 CONTINUE
C
C
      DO 18 I=1,NORDD
      AMAX(I)=AMAX1(AMAX(I),PRMT(I))
   18 AMIN(I)=AMIN1(AMIN(I),PRMT(I))
C
C
      WRITE(3,200)(PRMT(J),J=1,NORDD)
  200 FORMAT(1X,15F12.4)
C
C
      L=1
      DO 20 I=1,NDATA
      DO 21 J=1,NORD
      P(L)=P(L)+ERR*K((J-1)*NDATA+I)
   21 L=L+1
   20 CONTINUE
C
C     CALCULATION OF NEW COMER FOR SHIFT REGISTER "X"
C
      DO 25 I=1,NDATA
      S=0.0
      DO 26 J=1,NORD
   26 S=S+FV(J)*X((J-1)*NDATA+I)
      IF (I .EQ. 1)HIN=ERR*AH
      IF (I .EQ. 2)HIN=YU(KKK)
      IF (I .EQ. 3)HIN=YU(ITER+KKK)
   25 XI(I)=S+HIN
C
C  ***************************************************************
C  *  THE ITERATIVE METHOD OF CALCULATION OF "K" gain            *
C  *  BY Ljung IS USED. SEE Int. J. of Contr. Vol. 27 No 1       *
C  *  pp 1 19, 1978.                                             *
C  ***************************************************************
```



```
C
C     CALCULATION OF "EPSILON" OF Eqn. 4.1
C
      DO 30 I=1,NDATA
      S=0.0
      DO 31 J=1,NORDD
   31 S=S+A(J,I)*X(J)
   30 EPSI0(I)=XI(I)+S
C
C     UPDATE "A" OF Eqn. 4.2
C
      DO 33 I=1,NORDD
      DO 34 J=1,NDATA
   34 A(J,I)=A(J,I)-K(I)*EPSI0(J)
   33 CONTINUE
C
C     CALCULATION OF "EPSILON"  OF Eqn. 4.3
C
      DO 40 I=1,NDATA
      S=0.0
      DO 41 J=1,NORDD
   41 S=S+A(J,I)*X(J)
   40 EPSI(I)=XI(I)+S
C
C     UPDATE "ISIG" OF Eqn. 4.4. HERE THE MATRIX INVERSION
C     LEMMA HAS BEEN USED
C
      S=1.0
      DO 90 I=1,NDATA
      WKA(I,1)=0.0
      WKA(I,2)=0.0
      DO 91 J=1,NDATA
      WKA(I,1)=WKA(I,1)+ISIG(I,J)*EPSI(J)
   91 WKA(I,2)=WKA(I,2)+EPSI0(J)*ISIG(J,I)
   90 S=S+WKA(I,2)*EPSI(I)
C
      DO 95 I=1,NDATA
      DO 96 J=1,NDATA
   96 ISIG(I,J)=ISIG(I,J)-WKA(I,1)*WKA(J,2)/S
   95 CONTINUE
C
C     CALCULATION OF Kbar OF Eqn. 4.5
C
      DO 45 I=1,NDATA
      S=0.0
      DO 46 J=1,NDATA
   46 S=S+ISIG(I,J)*EPSI(J)
   45 WKA(I,1)=S
C
      DO 48 I=1,NORDD
      S=0.0
      DO 49 J=1,NDATA
   49 S=S+A(I,J)*WKA(J,1)
   48 K(I)=K(I)+S
```

Fig. 3-1 shows the geometric interpretation of equation (3.17). Now that we have obtained the optimal gain $a\Sigma/\lambda$ = we can go back to the original equation (3.7). With the choice of $\Sigma = S_I$ and $a = 1$, equation (3.7) can be written as:

$$\tilde{p}_i(k) = \tilde{p}_i(k) - \psi(k)\phi(k)'\tilde{p}_i(k), \quad i\in[1,n] \qquad (3.13)$$

where $\tilde{p}_i(k)'$ is the i-th row vector of $\tilde{P}(k)$.

We can see that for a given $\psi(k)$ the best possible scalar gain at each step is $\psi(k)'\tilde{p}_i(k)$.

The global asymptotic stability of equation (3.13) is guaranteed by the fact that condition (iv) in the K-N scheme implies that any n+r consecutive vectors $\phi(k)$ span the n+r dimensional Euclidean space, i.e.

$$[\phi(k), \phi(k+1), \ldots, \phi(k+n+r-1)] \quad \text{is nonsingular.}$$

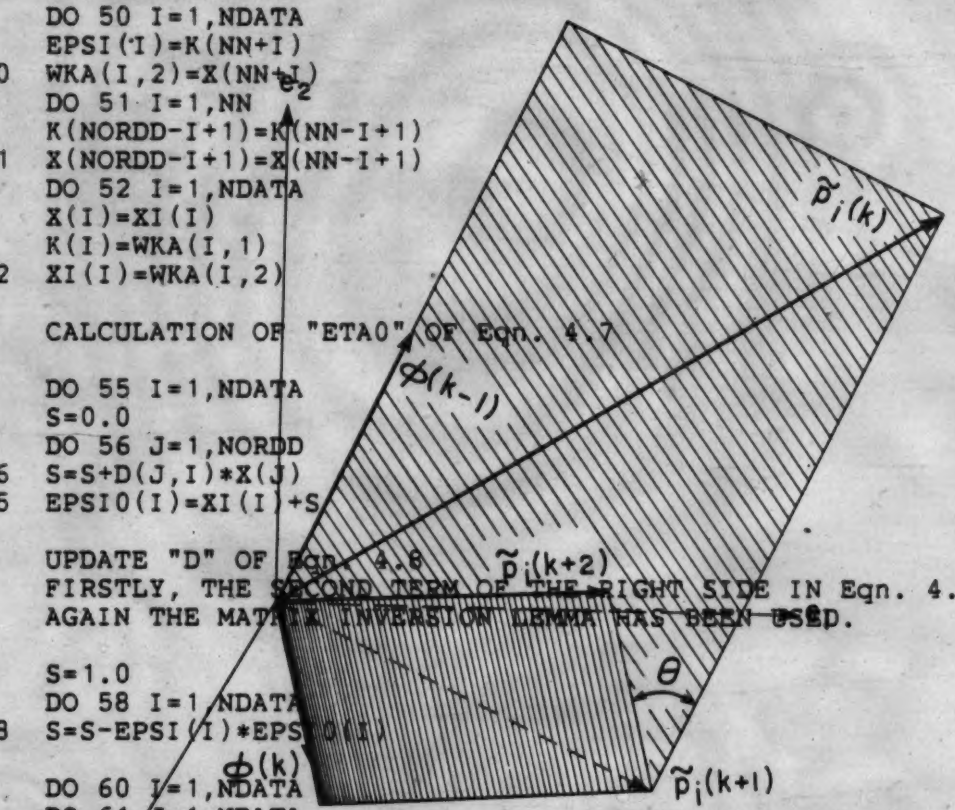
The implication of equation (3.13) is shown geometrically in Figure 3-2 for third order $\phi(k)$'s.

Keeping in mind the directions of $\phi(k-1)$ and $\psi(k)$ are the same, observation of Figure 3-2 reveals the following properties of $\tilde{p}_i(k)$:

a) $\tilde{p}_i(k+1)$ is orthogonal to $\psi(k)$,

b) $\tilde{p}_i(k+1)$ lies in the plane made up of $\tilde{p}_i(k)$ and $\psi(k)$

```
C
C  ***************************************************
C  *  HEREAFTER "EPSI0" WILL BE  USED AS "ETA0",     *
C  *            "XI" AS "XI(t-n)" in Eqn. 4.7        *
C  *            "EPSI" AS "MU" in Eqn. 4.6           *
C  *            "K" AS "M(t)" in Eqn. 4.6            *
C  ***************************************************
C
C     UPDATE "X" AND COMPUTE "M(t)" and "MU(t)" in Eqn. 4.6
C
      NN=NORDD-NDATA
      DO 50 I=1,NDATA
      EPSI(I)=K(NN+I)
   50 WKA(I,2)=X(NN+I)
      DO 51 I=1,NN
      K(NORDD-I+1)=K(NN-I+1)
   51 X(NORDD-I+1)=X(NN-I+1)
      DO 52 I=1,NDATA
      X(I)=XI(I)
      K(I)=WKA(I,1)
   52 XI(I)=WKA(I,2)
C
C     CALCULATION OF "ETA0" OF Eqn. 4.7
C
      DO 55 I=1,NDATA
      S=0.0
      DO 56 J=1,NORDD
   56 S=S+D(J,I)*X(J)
   55 EPSI0(I)=XI(I)+S
C
C     UPDATE "D" OF Eqn. 4.8
C     FIRSTLY, THE SECOND TERM OF THE RIGHT SIDE IN Eqn. 4.8
C     AGAIN THE MATRIX INVERSION LEMMA HAS BEEN USED.
C
      S=1.0
      DO 58 I=1,NDATA
   58 S=S-EPSI(I)*EPSI0(I)
C
      DO 60 I=1,NDATA
      DO 61 J=1,NDATA
      WKA(I,J)=EPSI(I)*EPSI0(J)/S
      IF (I .EQ. J)WKA(I,J)=WKA(I,J)+1.0
   61 CONTINUE
   60 CONTINUE
C
C     THEN, THE FIRST TERM OF THE RIGHT SIDE IN Eqn. 4.8
C
      DO 65 I=1,NORDD
      DO 66 J=1,NDATA
   66 D(I,J)=D(I,J)-K(I)*EPSI0(J)
   65 CONTINUE
C
C     COMBINE THE ABOVE TWO TERMS
C
```



```
      DO 70 L=1,NORDD
      DO 71 I=1,NDATA
      WKA(I,4)=0.0
      DO 72 J=1,NDATA
   72 WKA(I,4)=WKA(I,4)+D(L,J)*WKA(J,I)
   71 CONTINUE
      DO 73 M=1,NDATA
   73 D(L,M)=WKA(M,4)
   70 CONTINUE
C
C     UPDATE K#
C
      DO 75 I=1,NORDD
      S=0.0
      DO 76 J=1,NDATA
   76 S=S+D(I,J)*EPSI(J)
   75 K(I)=K(I)-S
C
C
 1000 CONTINUE
C
      WRITE(3,200)(AMAX(J),J=1,NORDD)
      WRITE(3,280)(AMIN(J),J=1,NORDD)
C
      WRITE(3,300)
  300 FORMAT(1X,'ESTIMATE OF C VECTOR IS FOLLOWED BY THAT
     $ OF A')
      WRITE(3,301)
  301 FORMAT(1X,' IN FORMAT F12.4 ')
      WRITE(3,302)
  302 FORMAT(' ')
      IF (ICH .EQ. 0)GOTO 303
      WRITE(3,304)
  304 FORMAT(1X,'THESE ARE OUTPUT OF ERF-M3')
      RETURN
  303 WRITE(3,305)
  305 FORMAT(1X,'THESE ARE OUTPUT OF GELS')
C
      RETURN
      END
```

3.3 A Rapidly Convergent Algorithm (Kim, et al., 1981)

While equation (3.13) maximizes the decrease of the parameter error $\tilde{p}_i(k)$ at each step and thus $\tilde{p}_i(k)$ is assured of convergence to the origin, the arbitrariness in the direction of $\psi(k)$ retards the speed of convergence $\tilde{p}_i(k)$ to the origin. In order to circumvent this difficulty, we introduce an orthogonal sequence of $\hat{\psi}(k)$ in the place of $\psi(k)$ to coordinate systematically the direction in which $\tilde{p}_i(k)$ decreases and adjust the scalar gain $\alpha(k)$ in such a way as to make $\tilde{p}_i(k+1)$ orthogonal to $\hat{\psi}(k)$ (This adjustable gain $\alpha(k)$ should not be confused with the fixed parameter α used in section 3.2). Then it is not difficult to see from the properties a) and b) that this n+r vector can be made to be parallel to $\psi(k)$ in n+r-1 steps. If this is done, then by property c), $\tilde{p}_i(k)$ will converge to the origin in one more step. Thus the total number of steps required to bring $\tilde{p}_i(k)$ to the origin will be utmost n+r.

Another way of explaining this is as follows. We choose $\psi(0)$ as the first basis vector of the n+r Euclidean space, $\hat{\psi}(0) = \psi(0)$. We select $\hat{\psi}(k)$ as the remaining basis vectors. We also choose $\alpha(k)$ to maximize $||\tilde{p}_i(k)|| - ||\tilde{p}_i(k+1)||$ at each step so that the number of steps required to bring $\tilde{p}_i(k)$ to the origin will be a minimum.

c) If $\tilde{p}_i(k)$ is parallel to $\psi(k)$ then $\tilde{p}_i(k+1)$ is zero, and $||\tilde{p}_i(k+1)|| < ||\tilde{p}_i(k)||$.

```
      SUBROUTINE RML(YU,FV,P,ITER,IYU,NORD,NORDD,GAIN,ICH)
C
C     THIS IS THE RECURSIVE MAXIMUM LIKELIHOOD
C     IDENTIFICATION ALGORITHM COMPLIMENTED WITH
C     PROJECTOR WHICH PROJECTS THE C ESTIMATES INTO THE
C     PREDETERMINED AREA FOR THE SYSTEM WITHOUT CONTROL
C     INPUT, i.e. TIME-SERIES OR WITH CONTROL INPUT.
C     THE RESULTS WILL BE STORED IN A TEMPORARY FILE "-RML"
C
C
C     YU:     OUTPUT OF THE SYSTEM WHOSE PAPRAMETERS ARE TO
C             BE IDENTIFIED AND THEN THE CONTROL INPUTS ARE
C             IMMEDIATELY FOLLOWED IF THEY EXIST.
C     FV:     THE VECTOR FV IS ALL ZERO FOR RML, THUS FORCED
C             TO BE ZERO.
C     P:      THIS VECTOR CONTAINS THE INITIAL ESTIMATES FOR
C             c, a and b VECTORS
C     ITER:   THE NUMBER OF ITERARTIONS REQUIRED, i.e. THE
C             LENGTH OF ARRAY OF Y
C     IYU:    THE NUMBER OF ENTRIES IN "YU" IF CONTROL
C             INPUTS EXIST THEN IYU=ITER*2,
C                     OTHERWISE IYU=ITER
C     NORD:   ORDER OF THE SYSTEM
C     NORDD:  2*NORD IF NO INPUTS
C             3*NORD IF THERE ARE INPUTS
C     GAIN:   SCALAR VALUE FOR THE INITIAL GAIN MATRIX P22
C             P22(0) WILL BE EQUAL TO GAIN*IDENTITY MATRIX
C     ICH:    ICH IS ADDED FOR THIS ROUTINE TO BE FORMAT
C             COMPATIBLE WITH GELS
C
C
      LOGICAL*1 DATA1(8)
      REAL F0(5,5),H(5),P22(15,15),UP(5,15),UN(5,15),
     $     Z1(5),Z2(5),Z3(5),PRMT(15),FV(15),S(15),T(5),
     $     YU(IYU),TMAX(15),TMIN(15),FV(NORD),P(NORDD),
     $     CINIT(5),EC(5,5)
      DATA H/1.0,4*0.0/
      DATA UP/75*0.0/
      DATA DATA1/'-','R','M','L',4*' '/
C
C
C     ATTACH THE PROPER TEMPORARY FILE FOR THE OUTPUTS
C
      CALL FTNCMD('ASSIGN 3=?;',0,DATA1)
C
C     DETERMINE IF THERE ARE INPUTS TO THE SYSTEM
C
      INPUT=0
      IF (NORDD .EQ. 3*NORD) INPUT=1
C
C     SET UP F0
C
      DO 600 I=1,NORD
```

In the light of the above discussion, we can use the following algorithm in place of equation (3.13)

$$\tilde{p}_i(k+1) = \tilde{p}_i(k) - u(k)\hat{\psi}(k)\phi(k-1)'\tilde{p}_i(k), \quad i\in[1,n] \qquad (3.14)$$

The choice of $u(k)$ to maximize $\|\tilde{p}_i(k+1) - \|\tilde{p}_i(k+1)\|\|$ is made as follows.

If an orthogonal sequence $\phi(k)$ is generated from the sequence of $\hat{\psi}(k)$ by the Gram-Schmidt process, the plane made up of $\hat{\psi}(k)$ and $\tilde{p}_i(k)$ and the plane made up of $\hat{\psi}(k)$ and $\phi(k-1)$ are orthogonal to each other (see Figure 3-3). $\tilde{p}_i(k)$ can be expressed as:

$$\tilde{p}_i(k) = \tilde{p}_i\hat{\psi}(k) + \tilde{p}_i\hat{\psi}(k)$$

where $\tilde{p}_i\hat{\psi}(k)$ is a projection of $\tilde{p}_i(k)$ on $\hat{\psi}(k)$ and $\tilde{p}_i\hat{\psi}(k)$ is a projection of $\tilde{p}_i(k)$ on a vector orthogonal to both $\hat{\psi}(k)$ and $\phi(k-1)$. Since $\phi(k-1)$ is orthogonal to $\tilde{p}_i\psi(k)$ and $\hat{\psi}(k)$, because of orthonormality, we can write

$$\tilde{p}_i(k) = \phi(k-1)'[\hat{\psi}(k)\hat{\psi}'(k)\tilde{p}_i(k)]$$

$$\text{INPUT=0} = [\phi(k-1)'\hat{\psi}(k)][\hat{\psi}(k)'\tilde{p}_i(k)] \qquad (3.15)$$

Using equation (3.15), we can write equation (3.14) as:

```
      DO 610 J=1,NORD
   P̃ᵢ(k+1) = P̃ᵢ(k) - α(k)ψ(k)[φ(k-1)'ψ̂(k)]ψ̂(k)'P̃ᵢ(k)   (3.16)
      IF(I .EQ. J+1) F0(I,J)=1.0
 610  CONTINUE
 600  CONTINUE
C
      DO 620 I=1,NORD
 620  FV(I)=0.0
C     Following equation (3.13), it has been pointed out that
C     SET INITIAL GAIN MATRIX P22(0)=GAIN*IDENTITY MATRIX
C     ψ̂(k)'P̃ᵢ(k) is the best possible scalar gain for a given
      DO 1 I=1,NORDD
C     ψ̂(k). Dividing by NORDD we see that the choice of
      DO 2 J=1,NORDD
      P22(I,J)=0.0
      IF (I .EQ. J) P22(I,J)=GAIN
   2  CONTINUE
   1  CONTINUE
C     α(k) = 1/[ψ̂(k)'φ(k-1)]                              (3.17)
C
C
      DO 5 I=1,NORD
C     in equation (3.16) will make ψ̂(k)'P̃ᵢ(k) the best possible
      Z2(I)=0.0
C     gain for any given ψ̂(k).
C
C     SET INITIAL ESTIMATES P,  P=[FV-C,A-FV,B]
C     The resulting equation:
      DO 7 I=1,NORD
      P(I)=FV(I)-P(I)
   7  P(I+NORD)=P(I+NORD)-FV(I)
C                          ψ̂(k)φ(k-1)'P̃ᵢ(k)
C  P̃ᵢ(k+1)=P̃ᵢ(k) - ------------------------[C,n]          (3.18)
C                          φ(k-1)'ψ̂(k)
      DO 8 I=1,NORD
   8  CINIT(I)=P(I)
C
C     DO 550 I=1,NORDD
C     is based on the same as the equation (3.13) except that the
      TMAX(I)=-100.0
C     direction of the sequence of P̃ᵢ(k+1) - P̃ᵢ(k) is now
C
C     orthogonal to that of all preceeding elements, which enables
C
C     P̃ᵢ(k)  DO convergence to the origin in utmost n+r steps.
C
C
C     The composite equation for all i, i∈[1,n] would be:
C
      DO 10 I=1,NORD
      FY(I)=Z1(I)
      FY(I+NORD)=Z2(P̃(k)φ(k-1)ψ̂(k)'
   P̃(k+1) (IF (INPUT) .EQ. 0) GOTO 10                    (3.19)
      FY(I+NORD*2)=Z3(ψ̂(k)'φ(k-1)
  10  CONTINUE
C
C
```

```
      YH=0.0
      DO 20 I=1,NORDD
   20 YH=YH+FY(I)*P(I)
C
C
      ERR=YU(K)-YH
C
C     CALCULATION OF FC = F0 - H*HTHETA2
C
      DO 40 I=1,NORD
      DO 42 J=1,NORD
   42 FC(I,J)=F0(I,J)-H(I)*P(J)
   40 CONTINUE
C
C     CALCULATION OF UP(k+1) in eqn. (4.24)
C
      DO 410 I=1,NORD
      DO 420 J=1,NORDD
      UN(I,J)=0.0
      DO 430 L=1,NORD
  430 UN(I,J)=UN(I,J)+FC(I,L)*UP(L,J)
  420 UN(I,J)=UN(I,J)-H(I)*FY(J)
  410 CONTINUE
C     CALCULATION OF S = P22(k)UP(k+1)'h in eqn. (4.28d)
C
      DO 450 I=1,NORDD
      S(I)=0.0
      DO 460 J=1,NORDD
  460 S(I)=S(I)+P22(I,J)*UN(1,J)
  450 CONTINUE
C     CALCULATION OF m1(k) with w=1 in eqn. (4.27)
C
      AM1=1.0
      DO 58 I=1,NORDD
   58 AM1=AM1+UN(J,I)*S(I)
      AM1=1.0/AM1
C     UPDATE Z1(k)
C
      DO 70 I=1,NORD
      T(I)=0.0
      DO 72 J=1,NORD
   72 T(I)=T(I)+F0(I,J)*Z1(J)
   70 T(I)=T(I)+H(I)*ERR
C
      DO 74 I=1,NORD
   74 Z1(I)=T(I)
C     UPDATE Z2(k)
C
      DO 80 I=1,NORD
      T(I)=0.0
```

Thus the resulting algorithm is:

$$P(k+1) = P(k) - \frac{P(k)\hat{\psi}(k)\hat{\psi}(k)'P(k)}{1 + \hat{\psi}(k)'P(k)\hat{\psi}(k)} \qquad (3.20)$$

Figure 3-3 shows the goemetric implication of equation (3.18).

The algorithm proposed by Westphal (1978) yields the same results as the algorithm (3.20) on if p in the Westphal's scheme is set equal to n+r-1. However, the emphasis placed by Westphal on the utilization of à priori information about the system parameters to improve the the convergence unnecessarily obscures the real strength of the algorithm. As the simulation results show in the next section, à priori information is not at all necessary for ensuring rapid convergence of the algorithm.

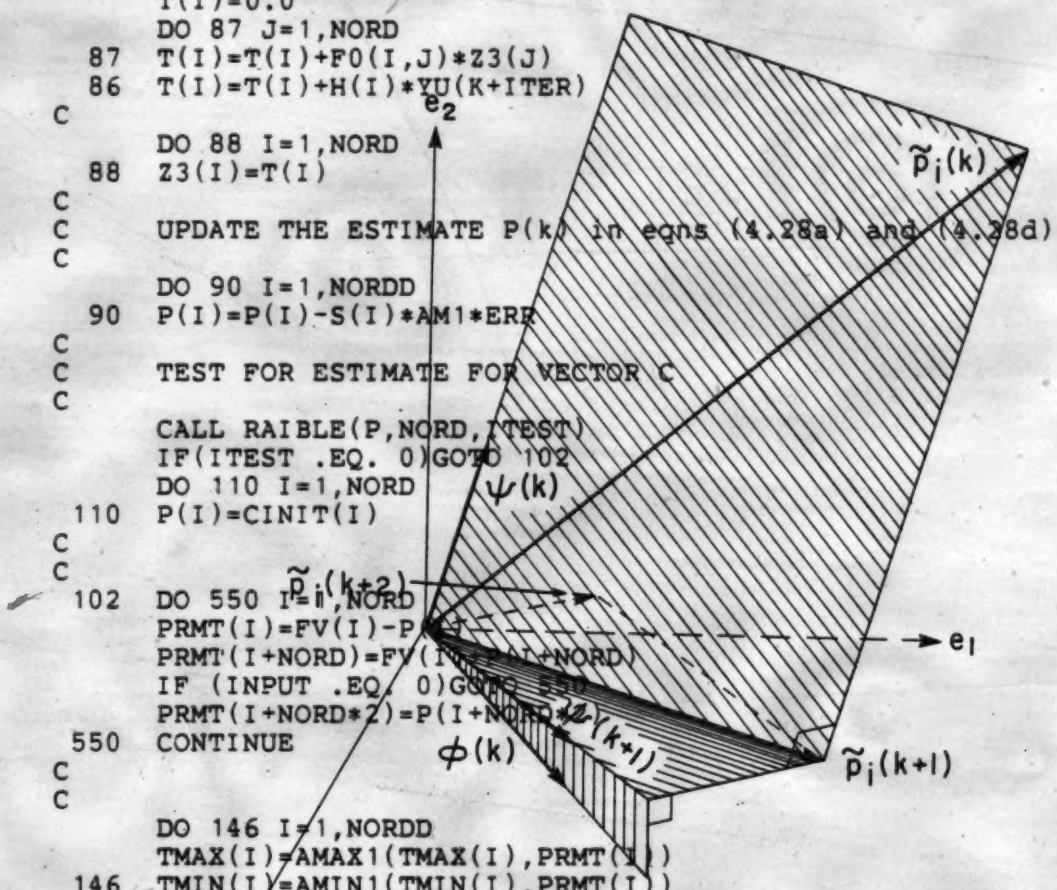
This algorithm is also readily applicable to a deterministic MIMO system whose present output can be described as an inner product of the parameters and a sequence of the past outputs and inputs.

3.4 Simulations

Using the algorithm (3.20), a computer simulation is first carried out to identify the parameters of the dynamical discrete system:

$$x(k+1) = Ax(k) + Bu(k)$$

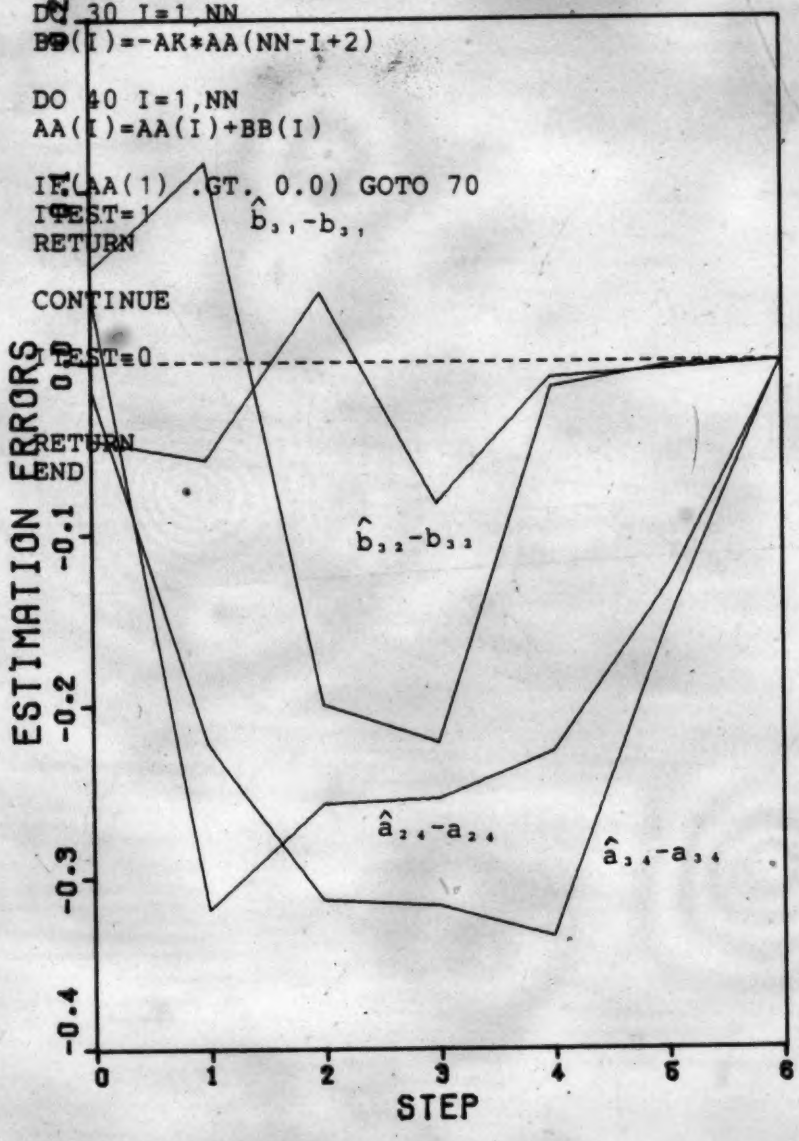
```
      DO 82 J=1,NORD
   82 T(I)=T(I)+F0(I,J)*Z2(J)
   80 T(I)=T(I)+H(I)*YU(K)
C
      DO 84 I=1,NORD
   84 Z2(I)=T(I)
C
C     UPDATE Z3(k)
C
      DO 86 I=1,NORD
      T(I)=0.0
      DO 87 J=1,NORD
   87 T(I)=T(I)+F0(I,J)*Z3(J)
   86 T(I)=T(I)+H(I)*YU(K+ITER)
C
      DO 88 I=1,NORD
   88 Z3(I)=T(I)
C
C     UPDATE THE ESTIMATE P(k) in eqns (4.28a) and (4.28d)
C
      DO 90 I=1,NORDD
   90 P(I)=P(I)-S(I)*AM1*ERR
C
C     TEST FOR ESTIMATE FOR VECTOR C
C
      CALL RAIBLE(P,NORD,ITEST)
      IF(ITEST .EQ. 0)GOTO 102
      DO 110 I=1,NORD
  110 P(I)=CINIT(I)
C
C
  102 DO 550 I=1,NORD
      PRMT(I)=FV(I)-P
      PRMT(I+NORD)=FV(I+NORD)-P(I+NORD)
      IF (INPUT .EQ. 0)GOTO 550
      PRMT(I+NORD*2)=P(I+NORD*2)
  550 CONTINUE
C
C
      DO 146 I=1,NORDD
      TMAX(I)=AMAX1(TMAX(I),PRMT(I))
  146 TMIN(I)=AMIN1(TMIN(I),PRMT(I))
C
C
      WRITE(3,200)(PRMT(J),J=1,NORDD)
  200 FORMAT(1X,15F12.4)
C
C     UPDATE THE GAIN MATRIX P22(k)
C
      DO 120 I=1,NORDD
      DO 122 J=1,NORDD
  122 P22(I,J)=P22(I,J)-S(I)*S(J)*AM1
C
C     MOVE U(k+1) TO U(k)
```



3.3 The geometric effect of orthonormal vectors

```
C
      DO 124 L=1,NORD
  124 UP(L,I)=UN(L,I)
  120 CONTINUE
C
C
C
C
  777 CONTINUE
C
C
      WRITE(3,200)(TMAX(J),J=1,NORDD)
      WRITE(3,200)(TMIN(J),J=1,NORDD)
C
C A =
      WRITE(3,300)
  300 FORMAT(1X,'ESTIMATE OF C VECTOR IS FOLLOWED BY
     $ THAT OF A')
      WRITE(3,301)
  301 FORMAT(1X,'IN FORMAT F12.4')
      WRITE(3,302)
  302 FORMAT(' ')
C
      WRITE(3,305)
  305 FORMAT(1X,'THESE ARE OUTPUT OF RML')
C
      RETURN
      END
C
C
C
C
```

```
      AA(1)=1.0
C
      DO 70 K=1,N
      AK=AA(NN)/AA(1)
      NN=NN-1
C
      DO 30 I=1,NN
   30 BB(I)=-AK*AA(NN-I+2)
C
      DO 40 I=1,NN
   40 AA(I)=AA(I)+BB(I)
C
      IF(AA(1) .GT. 0.0) GOTO 70
      ITEST=1
      RETURN
C
   70 CONTINUE
C
      ITEST=0
C
C
      RETURN
      END
```



3.4 The results of the first simulation